Precision Predictions at N^{3}LO for the Higgs Boson Rapidity Distribution at the LHC

based on 1810.09462,
with Falko Dulat and Bernhard Mistlberger

A. Pelloni

18.04.2019

OUTLINE

- Motivations
- Theoretical Framework
- Rational Reconstruction
- Threshold Expansion
- Reaching Beyond Threshold
- Results

MOTIVATIONS

- The LHC target luminosity is 3000 fb^{-1}, this will reduce the experimental uncertainty

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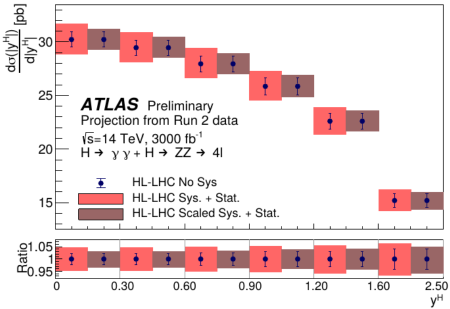


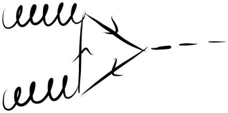
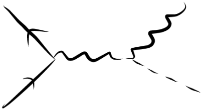
MOTIVATIONS

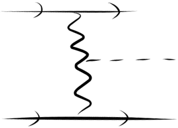
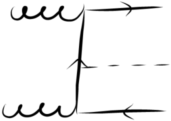
- The LHC target luminosity is 3000 fb^{-1}, this will reduce the experimental uncertainty
- Beginning of transition from observation to precise measurement has just started
- Differential cross section means flexibility for phenomenology (e.g. compute decays)
- Crucial to providing precise predictions to test and find new physics!
- Check stabilization of the perturbative expansion of the rapidity distribution, as for the inclusive **N3LO** .

Hard challenge!

- Differential translate in more variables, this becomes a challenge when manipulating analytic expressions
- Simple reduction to master integrals will fail. The Coefficients of the reductions become massive.
- Need to use new techniques compared to the inclusive at the same order.

PRODUCTION CHANNELS

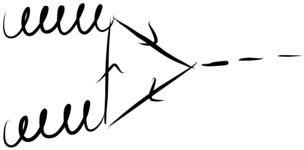
ggF	VH
 88.2%	 4.1%

VBF	$t\bar{t}H$
 6.8%	 0.9%

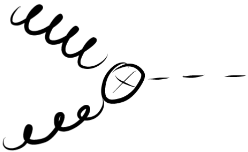
INFINITE TOP MASS

The process that we are looking at is the Higgs production via gluon fusion, computed in the infinite top mass limit.

Effective theory:



$$\xrightarrow{EFT}$$



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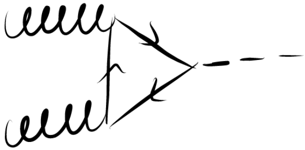

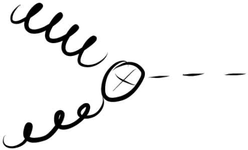

- Remove one loop!
- Good approximation: $\delta_t^{NNLO} \sim 0.7\%$
- To be combined with mass corrections, EWK corrections, etc...

HIGGS DIFFERENTIAL

We want to compute the differential cross section for the Higgs production:

The real radiation is integrated out, we are left with the partonic Higgs-differential x-section:

$$\frac{d^2\hat{\sigma}_{ij\to H+X}}{dY\,dp_T^2} \sim \int d\phi_n \left|\mathcal{M}_{ij\to H+X}\right|^2$$

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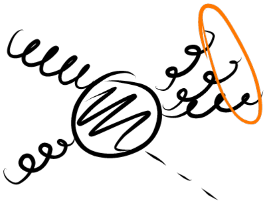

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Rapidity Distribution

$$\frac{d\hat{\sigma}_{ij\to H+X}}{dY} = \int dp_T^2 \frac{d^2\hat{\sigma}_{ij\to H+X}}{dY\,dp_T^2}$$

RAPIDITY DISTRIBUTION

The general form of the rapidity distribution can be written as:

$$\frac{d\sigma_{PP\to H+X}}{dY} = \hat{\sigma}_0 \sum_{ij} \int_0^1 dx_1 dx_2 dy_1 dy_2 f_i(y_1) f_j(y_2) \delta(\tau - x_1 x_2 y_1 y_2) \times \delta\left(Y - \frac{1}{2}\log\left(\frac{x_1 y_1}{x_2 y_2}\right)\right) \eta_{ij}(x_1, x_2),$$

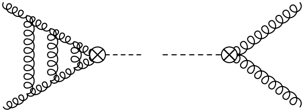
Where we define the partonic cross section

$$\eta_{ij}(x_1, x_2) = \sum_{k=0}^{3} \left(\frac{\alpha_S}{\pi}\right)^k \eta_{ij}^{(k)}(x_1, x_2).$$

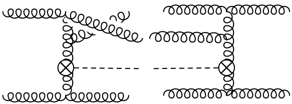
INTERFERENCES

Many contributions to be considered:

VVV + Born:



RRR + RRR:



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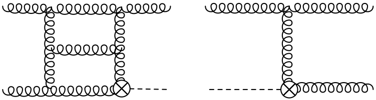

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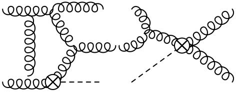


. . .

ROAD TO COMPUTATION

One of the standard tools to be used to resize the magnitude of the problem is to identify by means of Integration By Part (IBP) identities a set of Master integrals to span the space of the scalar integrals that appear in the computation:

$$F\left(s_{ij},\epsilon\right) = c_i\left(s_{ij},\epsilon\right) M_i\left(s_{ij},\epsilon\right)$$

- c_i: Coefficient that depends on the external kinematics together with the dimensional regulator ϵ.

  Rational Functions

- M_i: Master integrals (i.e. scalar integrals) that depend on the external kinematics together with the dimensional regulator.

  Special Functions (Multiple Polylogarithms, Elliptic Functions,...)

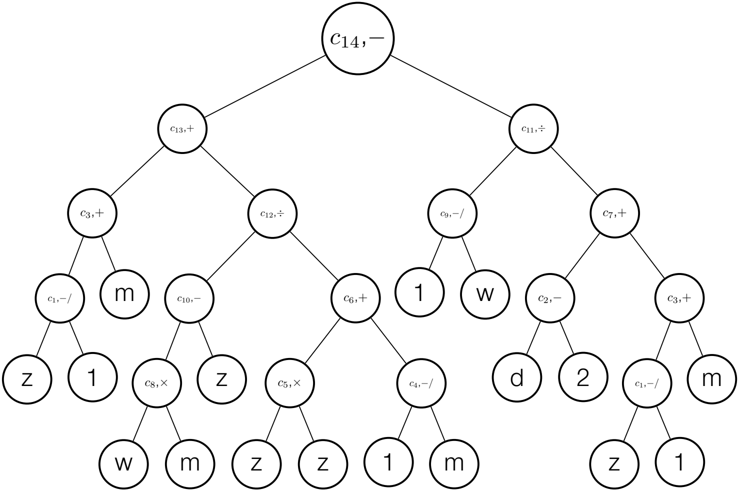
IBP

We can reduce the Integrals to be computed to Master Integrals through the Laporta algorithm. However, it's not that simple. Differential means more variables in the final answer:

- Symbolic reduction using Laporta Algorithm: FAST
- Algebraic evaluation of the reduction coefficients: SLOW

IBP

Reduction coefficients are stored in trees:



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Solution: Evaluate the coefficients numericallys and then infer from these evaluations the analytic expression.

RATIONAL FUNCTION RECONSTRUCTION

With enough evaluations, it's always possible to understand the structure of any rational function.

$$f(t) := \frac{p(t)}{q(t)}, \qquad rank(p) = r_n, \quad rank(q) = r_d.$$

With $n = 2\max\{r_n, r_d\} + 1$ evaluations we can reconstruct the function above by means of Thiele's interpolation formula:

$$\frac{p(t)}{q(t)} = a_1 + \cfrac{(t-t_1)}{a_2 + \cfrac{(t-t_2)}{a_3 + \cfrac{(t-t_3)}{a_4 + \cdots}}}$$

MULTI-VARIABLE FUNCTIONS

There is no general way to reconstruct a rational function with more than one variable because of non-trivial singularities and accidental cancellations.
It's possible if we assume the following canonical form:

$$f(x_1, ..., x_N) = \frac{\sum_{p=0}^{r_n} a_p \mathbf{x}^{\alpha_p}}{1 + \sum_{q=1}^{r_d} b_q \mathbf{x}^{\beta_q}},$$

$$\mathbf{x}^{\alpha_p} := \prod_{i=1}^{N} x_i^{\alpha_p^i}, \quad \alpha_p := \{\alpha_p^1, ..., \alpha_p^N\}, \qquad |\alpha_p| = p.$$

MULTI-VARIABLE FUNCTIONS

Algorithm (sketch):

1. Pick a set of random points $\{x_1^0, ..., x_N^0\}$ where $f(x^0)$ is not singular
2. Rescale them by t and reconstruct $g(t) := f(t \cdot x_1^0, ..., t \cdot x_N^0)$
3. Now the coefficients a_p and b_q are **polynomial** in $\{x_1, ..., x_N\}$ evaluated at $\{x_1^0, ..., x_N^0\}$
4. Repeat point 1 and 2 to obtain enough evaluations to reconstruct the polynomial functions a_p and b_q.

EXAMPLE

Let's try to reconstruct the function: $f(x,y) := \frac{x+y}{1+x}$.

- We do the reconstruction for $(x_0, y_0) = (1,1)$ by evaluating $f(tx_0, ty_0)$ for $t = 1,2,3$. The solution is: $f(tx_0, ty_0) = \frac{2t}{1+t}$
- We pick a new set of points $(x_1, y_1) = (1,2)$ and we evaluate $f(tx_1, ty_1)$ for $t = 1,2,3$. The solution is: $f(tx_1, ty_1) = \frac{3t}{1+t}$
- We now know that our final answer looks like:

$$f(tx, ty) := \frac{c(x,y)t}{1 + d(x,y)t}$$

where $c(x,y)$ and $d(x,y)$ are homogeneous functions in x and y. From the two reconstructions we have:

$$\left.\begin{array}{l} c(1,1) = 2 \\ c(1,2) = 3 \end{array}\right\} \Rightarrow c(x,y) = x + y, \qquad \left.\begin{array}{l} d(1,1) = 1 \\ d(1,2) = 1 \end{array}\right\} \Rightarrow d(x,y) = x.$$

PROBLEMS

In order to be able to recover the structure of the rational(polynomial) functions we need to work over rational numbers of arbitrary precision.

$\rightarrow$ Numbers in intermediate steps are BIG

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Another dead end?

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Another dead end?

NO.
Still possible to evaluate in short time if we work with machine size integers!

FINITE FIELD

- Well known techniques that are just waiting to be used:
  - $\rightarrow$ CRT (Sun Tsu, 3rd-century ad)
  - $\rightarrow$ RR (Wang, 1981-1982)
- Becoming more and more popular in high energy physics since their first applications. **[A. von Manteuffel and R. M. Schabinger ’15]**

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- Easily parallelizable
- Great improvement in performances especially because the final answer contains relatively small numbers.

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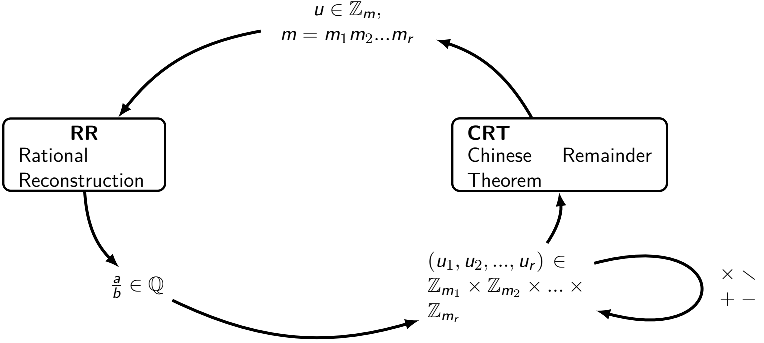
e.g: If the coefficients are of machine size (32bits) we require just three evaluations:

$$m_1 = 5817113, \quad m_2 = 4869863, \quad m_3 = 2015177$$

RATIONAL RECONSTRUCTION



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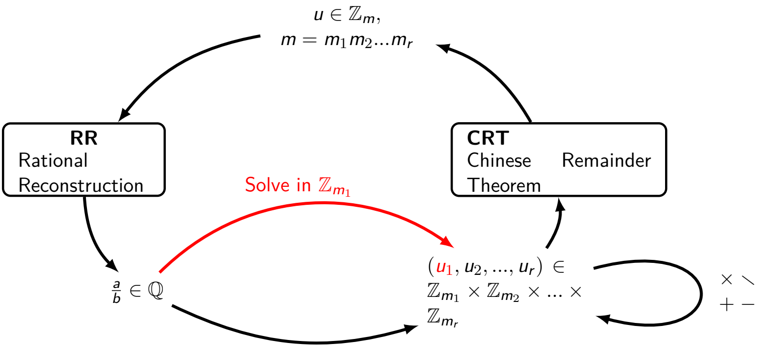

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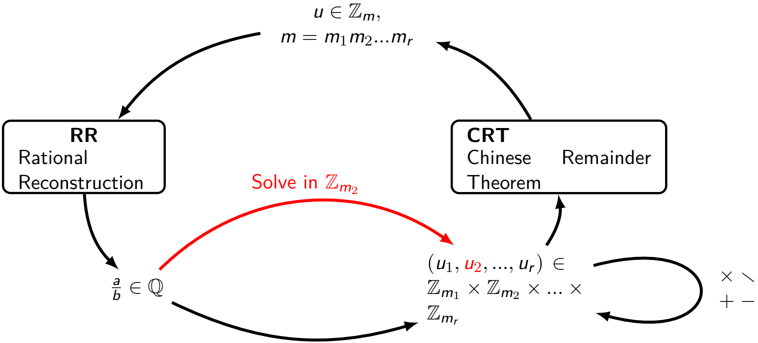

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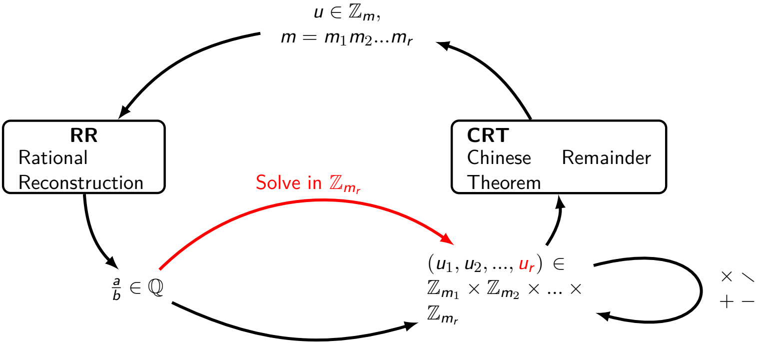

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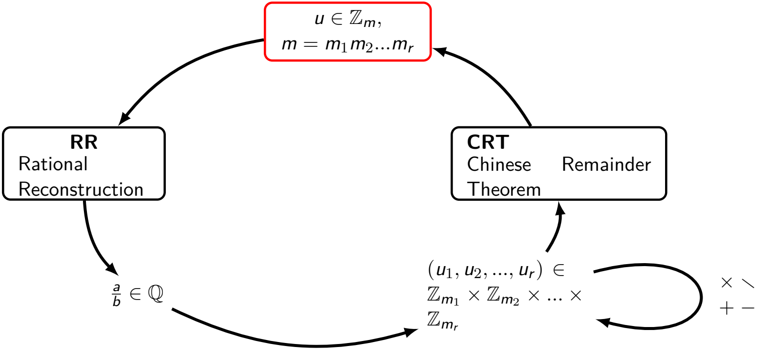

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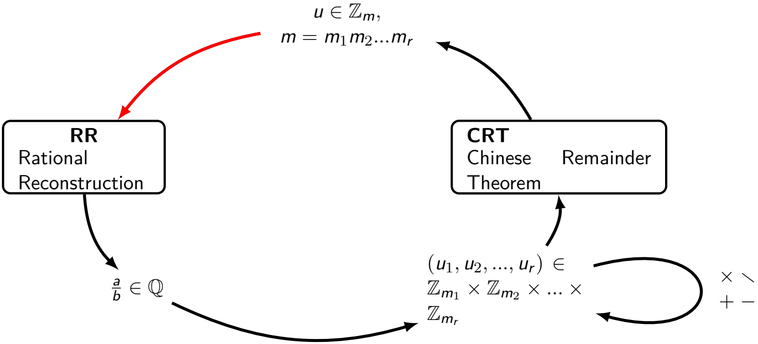

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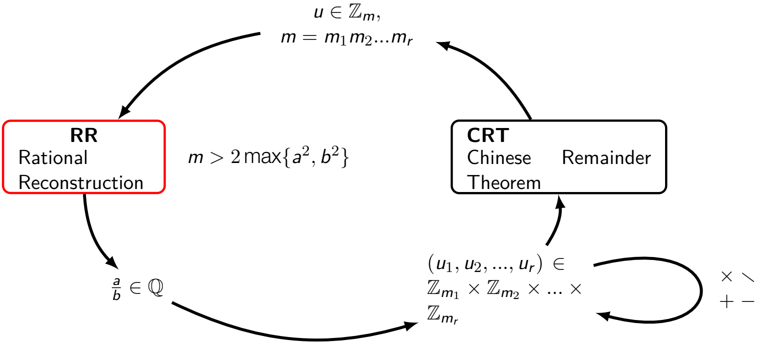

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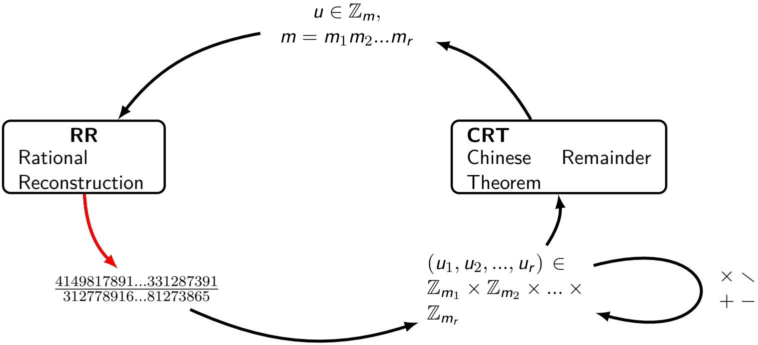
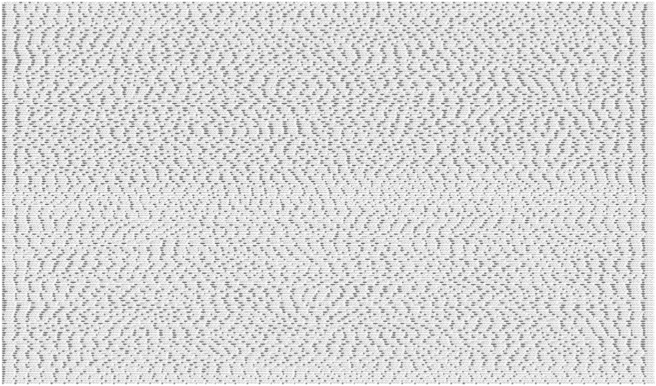


EXAMPLE

Reconstructed expression with $\sim 100'000$ non-zero coefficients



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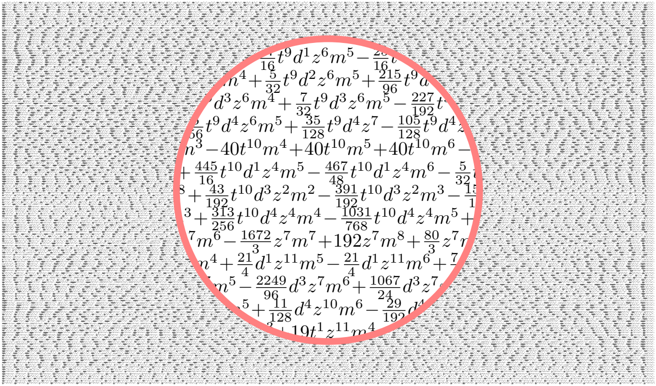


EXAMPLE

Reconstructed expression with $\sim 100'000$ non-zero coefficients

- Worst coefficient for this topology had $\sim 30'000'000$ non zero coefficients
- Because of the shift this number translate in a higher number of numerical coefficients that need to be reconstructed

$$t^n \to (t - t_0)^n$$

- Possible to reconstruct all the coefficients within a week!

ROAD TO COMPUTATION II

Computing the analytic result for the analytic rapidity distribution is a hard challenge!

$$\frac{d\hat{\sigma}_{ij\to H+X}}{dY}$$

ROAD TO COMPUTATION II

Computing the analytic result for the analytic rapidity distribution is a hard challenge!

Divide and Conquer

$$\frac{d\hat{\sigma}_{ij\to H+X}}{dY}$$

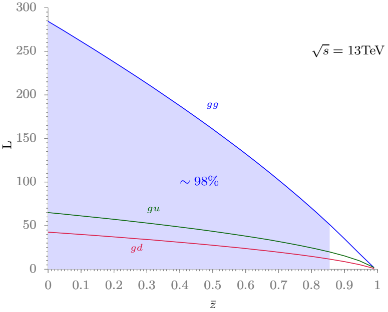
- Perform expansion around the production threshold. Already a success for the inclusive **N3LO**

$$\bar{z} = 1 - z = 1 - \frac{m_H^2}{s} \sim 0$$

- Expand to sufficiently higher orders

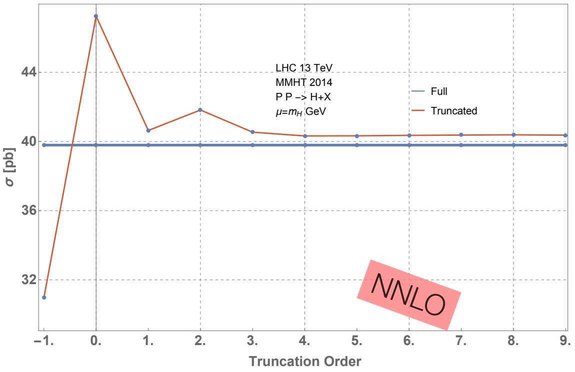
PARTON LUMINOSITY

$$L(z) = \int_{\frac{\tau}{z}}^{1} \frac{dx}{x} f_i(x) f_j\left(\frac{\tau}{z\,x}\right).$$



- The probability of producing the Higgs boson as a function of the partonic center of mass is reduced as the energy moves away from the threshold

THRESHOLD EXPANSION



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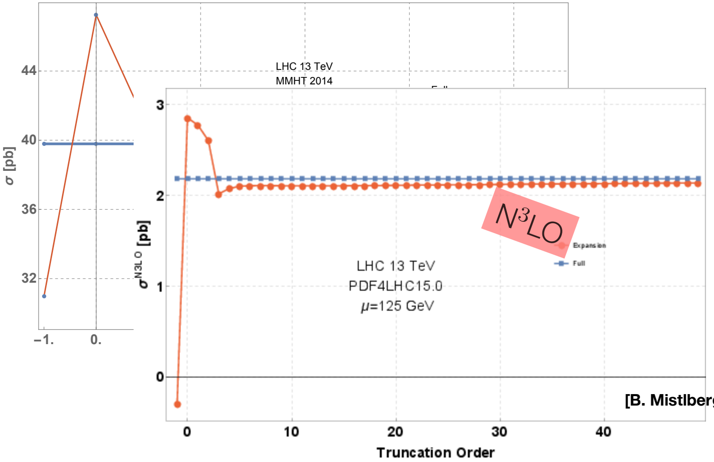
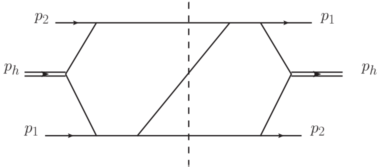


[B. Mistlberger ’18]

CROSS SECTION EXPANSION

Consider the case where there are only real corrections, $RRR + RRR$:

$$I(p_1, p_2, k) \quad = $$



$$= \quad \int \mathrm{d}\Phi_3 \frac{1}{p_{23}^2 p_{25}^2 p_{34}^2 p_{45}^2 p_{134}^2 p_{145}^2},$$

Where $p_{i_1 \dots i_n} = p_{i_1} + \dots + p_{i_n}$ and $k = p_{345}$.

- Threshold limit correspond to the limit where all radiation produced in association with the Higgs is uniformly soft

$$p_{3,4,5} \to \bar{z}\, p_{2,3,4}, \quad \mathrm{d}\Phi_3 \to \bar{z}^{2d-6} \mathrm{d}\Phi_3$$

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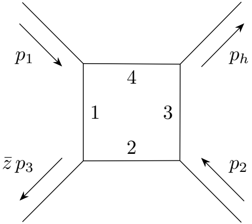
LOOP MOMENUTM

The loop momentum can take arbitrarily small and large values compared to the parameter $\bar{z}$. We need to split the expansion into different sectors!



$$A_1 = l^2,\; A_2 = (l - \bar{z}\,p_3)^2,\; A_4 = (l - p_1)^2$$

$$A_3 = (l - \bar{z}\,p_3 + p_2)^2$$

Naive expansion only converges for large values of the loop momentum (Hard sector):

$$\frac{1}{(l - \bar{z}\,2\,l \cdot p_3)^2} = \frac{1}{l^2} \sum_{n=0}^{\infty} \left(\frac{\bar{z}\,2\,l \cdot p_3}{l^2} \right)^n$$

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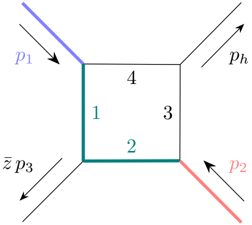

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C_1: $l_1^2 \sim \bar{z} \quad l_1 \cdot p_1 \sim 1 \quad l_1 \cdot p_2 \sim \bar{z}$

C_2: $l_2^2 \sim \bar{z} \quad l_2 \cdot p_1 \sim \bar{z} \quad l_2 \cdot p_2 \sim 1$

S: $l_1 \sim \bar{z}$

THRESHOLD EXPANSION

In dimensional regularization the expression for the partonic cross section takes the form,

$$\eta_{ij}^{(3)}(x_1, x_2) = \eta_{ij}^{(3)}\delta(1-x_1)\delta(1-x_2) + \sum_{n,m=1}^{3} (1-x_1)^{-1-m\epsilon}\,(1-x_2)^{-1-n\epsilon}\,\eta_{ij}^{(3,m,n)}(x_1, x_2),$$

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- Different sectors of the loop momentum give rise to different m ,n exponent
- $m = 1$ or $n = 1$ are known exactly! $\leftarrow$ Genuine two loop contributions

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$$\eta_{ij}^{(3)}(x_1,x_2) = \eta_{ij}^{(3)}\delta(1-x_1)\delta(1-x_2) + \sum_{n,m=1}^{3} \underbrace{(1-x_1)^{-1-m\epsilon}(1-x_2)^{-1-n\epsilon}}_{\text{Distributions}} \underbrace{\eta_{ij}^{(3,m,n)}(x_1,x_2)}_{\text{Holomorphic}},$$

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DISTRIBUTION

We can extract the divergence by means of the dimensional regulator ϵ obtaining a combination of distribution, in particular δ-functions and plus-distributions:

$$\int_0^1 dx (1-x)^{-1+a\epsilon} f(x) = \int_0^1 dx \frac{f(x) - f(1)}{(1-x)^{1-a\epsilon}} + \int_0^1 dx \frac{f(1)}{(1-x)^{1-a\epsilon}}$$

$$= \int_0^1 dx \left[\frac{\delta(1-x)}{a\,\epsilon} + \sum_{n=0}^{\infty} \frac{(a\,\epsilon)^n}{n!} \left[\frac{\log^n(1-x)}{1-x} \right]_+ \right] f(x)$$

With $f(x)$ some test function.

REACHING BEYOND THRESHOLD EXPANSION

Obtain finite expressions with a suitable mass factorization and ultraviolet renormalization counter term $CT_n^{(3)}$:

$$\eta_{ij}^{(3)}(x_1, x_2) = \lim_{\epsilon \to 0} \left[\eta_{ij,bare}^{(3)}(x_1, x_2) + CT_{ij}^{(3)}(x_1, x_2) \right]$$

- Use the fact that poles in the dimensional regulator ϵ cancel to impose further constraints on the PCF
- Fix most of the logarithmically enhanced terms
- Smaller set of expressions that need threshold expansion

MATCH TO THE INCLUSIVE

Integrate over the rapidity to recover the inclusive x-section,

$$\eta_{ij}^{(3),incl.}(z) = \int dY \eta_{ij}^{(3)}(x_1, x_2).$$

- Strong check on the differential partonic cross section
- Agreement between the two threshold expansions for all computed orders!

MATCHING THE INCLUSIVE

- We have 6 terms in the threshold expansion!
- Impose conditions to the missing orders in $\bar{z}$ such that it matches the inclusive at all orders!

$$\eta_{ij}^{(3),matched}(x_1, x_2) = \eta_{ij}^{(3),app.}(x_1, x_2) + \frac{x_1 + x_2}{2(1 - x_1 x_2)} \times \left[\eta_{ij}^{(3),inc.}(x_1 x_2) - \eta_{ij}^{(3),inc.,app.}(x_1 x_2)\right], \quad (1)$$

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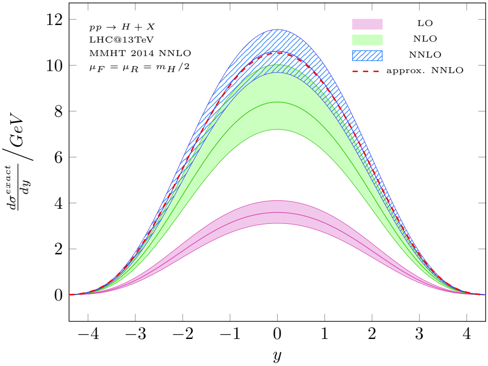
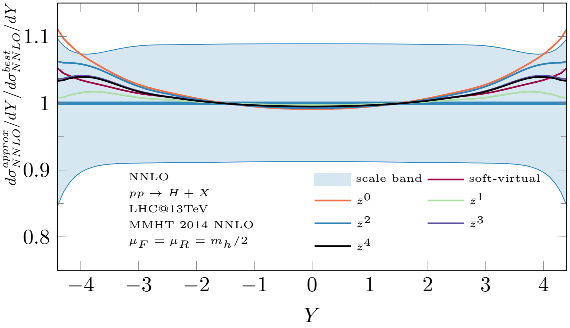
THRESHOLD AT NNLO

Applying the threshold expansion to NNLO gives good approximations:

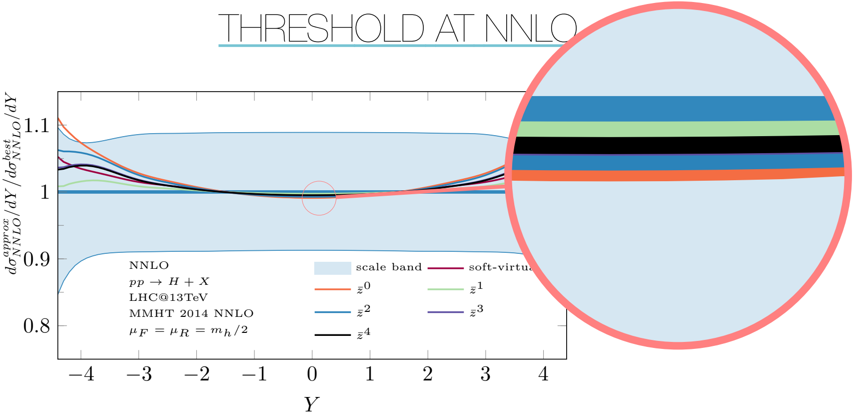


THRESHOLD AT NNLO



- The approximation performs well for central rapidities $|Y| < 3$
- Consistent improvement by including more terms
- To access the missing information from high energy contribution and fill the gap to the exact NNLO we need other tools.

THRESHOLD AT NNLO



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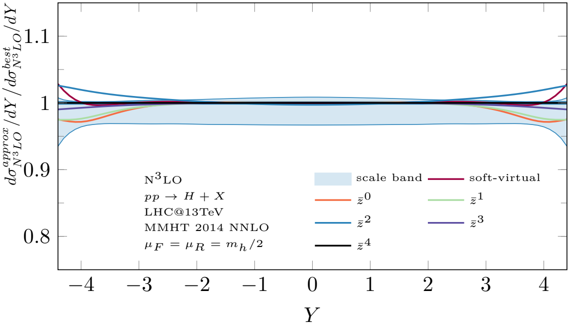
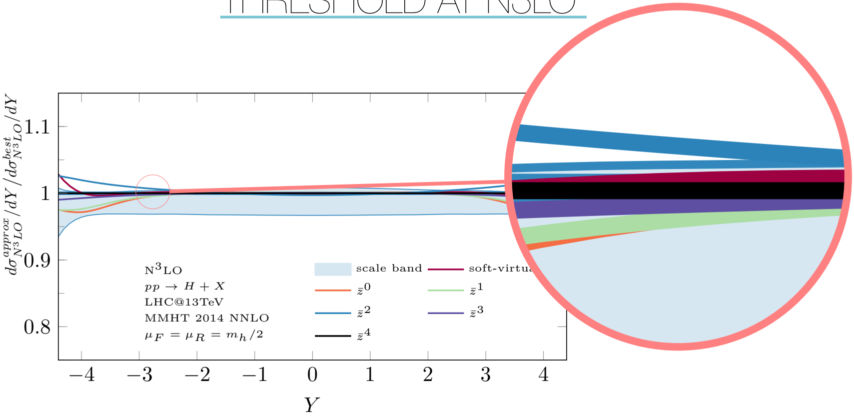
THRESHOLD AT N3LO



- Consistent behaviour between NNLO and **N3LO** regarding threshold expansion!
- Large rapidities show more variation

THRESHOLD AT N3LO



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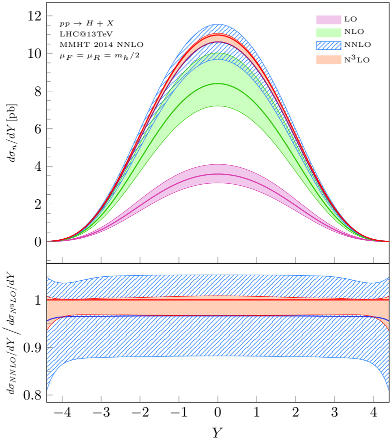
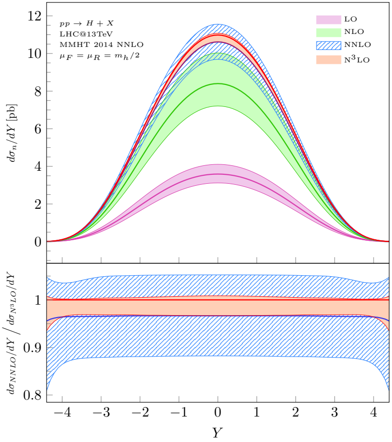
RAPIDITY



- The **N3LO** correction is well within the scale variation of NNLO!
- Significant reduction of scale uncertainty [-3.4%,+0.9%]
- Agreement with another approximation

[Cieri,Chen,Gehrmann,Glover,Huss]

RAPIDITY



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Flat !

CONCLUSION

- We computed the Higgs boson rapidity distribution at **N3LO**
- We observe stabilisation of perturbative correction and a significant reduction in the variation of the cross section as a function of the perturbative scale.
- **N3LO** corrections are uniform throughout the entire rapidity range.
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Thank you!