Open Problems in Planet Formation | Phil Armitage
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• Observational context
  - exoplanets
  - imaging of protoplanetary disks
• Back-of-the-envelope physics
  - aerodynamics, gravity, stability
• Some open problems
  - how did the first "planetesimals" form
  - (predicting long-term dynamical stability)

Research work with Jake Simon (Iowa State), Shirley Ho (CCA)
Key facts about the Solar System

Planets have low eccentricity, low inclination orbits, with bulk of the Solar System angular momentum in the planets not the Sun (mostly in Jupiter’s orbit) → Nebula Hypothesis

Architecture
Inner terrestrial planets (“rocky”, secondary atmospheres)
Outer giant planets (10-300x more massive, envelopes)
Various small bodies

Giant planets are mostly H/He, but are enriched in heavier elements relative to Solar composition. Possess rocky or icy cores (exact nature of Jupiter’s core is open issue)

System is highly chaotic (loosely: Lyapunov time ~10 Myr) but nonetheless quasi-stable… O(1%) chance of major instability (planetary collision) in next 5 Gyr

“Classical” theory of planet formation (Safronov, Wetherill…) largely self-consistent model that broadly explains these properties
Exoplanets

![Diagram showing planet mass vs. separation in Astronomical Units (AU).]

- **Planetary Mass [Jupiter Mass]**
- **Separation [Astronomical Units (AU)]**

*Note: The graph illustrates the distribution of exoplanets with various masses and separations from their host stars.*
• Exoplanets are common (order 1 detected per star)
• Both planets and planetary systems exhibit diversity that is unexpected based on Solar System expectations
Exoplanets

Giant planet sample:

- “hot Jupiters” $a < 0.1$ AU
- broad eccentricity distribution at radii where tidal effects negligible $<e> \sim 0.2$
Exoplanets

hot Jupiters: orbital angular momentum not always aligned to the spin of the host star
Exoplanets

Bimodal distribution of planet radii
Most common type of detected planet has a radius intermediate between rocky and giant Solar System planets
Exoplanets

Inferred density is interpreted as suggesting a transition between “rocky” composition and a composition that includes a massive has envelope at a few x Earth mass.
Protoplanetary disks

- Disks of gas + solid particles ("dust") in approximate rotational equilibrium around young stars
- Lifetime \( \sim 3 \times 10^6 \) yr
- Masses \( 10^{-3} \) – \( 10^{-1} \) \( M_\odot \)
- 99% gas (H/He), 1% solids
- Density \( \sim 10^{-9} \) g cm\(^{-3}\) @ 1AU
- \( T \sim 10^3 \) – 10 K

Solid material is initially in form of small \( \sim \mu \text{m} \) particles

HL Tau, image in mm-wavelength thermal radio emission from dust
Classical theory

Assume that we first form **planetesimals** (~km-scale bodies) with a smooth surface density as function of orbital radius:

\[ \Sigma_p \propto r^{-3/2} \]

Planetesimals evolve under gravity (only), collide and coagulate

Initially a coagulation problem

1. **Initial runaway growth**
2. “Oligarchic” growth once largest bodies excite the random velocities of nearby planetesimals
Final assembly of terrestrial planets treated with N-body simulations (N finite radius masses interacting under gravity + collisions)

Initial conditions with a few Earth masses of bodies between 0.5-2 AU evolve to reasonable approximations of the Solar System

Simulation: Sean Raymond
Interesting discrepancies (e.g. low mass of actual Mars) but overall a very simple growth model starting from planetesimals reproduces much of what we see… “pure gravitational dynamics”
Final outcome from stability considerations

2 planets on initially circular orbits are stable if separated by $C.R_H$

$$R_H = \left( \frac{m}{3M} \right)^{1/3} a$$

$N$ planets become unstable on timescale that is exponential in initial separation

$$t_{\text{inst}} \propto \exp[\Delta/(m/M)^\gamma]$$

$\gamma = \frac{1}{4} - \frac{1}{3}$

Fixed mass in planets becomes more (quasi)-stable as collisions proceed and $N$ reduces

*Obertas+ (2017)*
Giant planet formation

Core (5-20 Earth masses) forms while gas is still present (few Myr), core captures a primordial envelope from the protoplanetary disk

Why?

- Mass in planetesimals in an annulus of width $R_H$ is an increasing function of orbital radius
- Where $T < 150K$ water is in the form of ice rather than vapor... more solid material available

$$R_H = \left( \frac{m}{3M} \right)^{1/3} a$$

Models suggest Jupiter can form within a few Myr

Faster if some of the solid mass is accreted in the form of small solids ("pebbles") rather than planetesimals
Extrasolar planets
largest scale that we directly observe

“How do planetesimals form?”

“planetesimals”
$\Sigma_p(r)$ effective initial conditions for subsequent growth – predictable from first principles?

observable in least collisionally evolved environments in the Solar System?
why a problem?

simplest hypothesis
adhesive pairwise collisions

how the mm-sized solids form
observed in chondrules
collision rate is fast enough to
grow larger objects
why a problem?

inspiral $\sim 10^3$ yr

fast aerodynamic loss toward star

collision velocities exceed fragmentation threshold

$\mathbf{s \sim cm-m}$

Blum & Wurm (2008)
solid-gas interaction

radius \( s \), mass \( m \)

relative velocity \( \Delta v \)

aerodynamic coupling in Epstein regime (\( s < \lambda \))

\[
F_{\text{drag}} = -\frac{4\pi}{3} \rho s^2 v_{\text{thermal}} \Delta v
\]

For a single particle define:

- **stopping time**
  \[
t_s = \frac{m \Delta v}{F_{\text{drag}}} = \frac{\rho m}{\rho} \frac{s}{v_{\text{thermal}}}
\]

- **dimensionless stopping time**
  \[
  \tau_s \equiv t_s \Omega_K
  \]
solid-gas interaction

single solid particle:
- drifts radially at $v_r(\tau)$
- has azimuthal velocity that differs from gas

“fluid” of solid particles has equilibrium drift that depends on:
- coupling $\tau$
- speed of the gas compared to Kepler speed $\eta v_K$
- ratio of the surface density of solids to gas $Z$
streaming instability

model system of drifting particles as an incompressible gas coupled aerodynamically to a compressible particle fluid

\[
\frac{\partial \rho_p}{\partial t} + \nabla \cdot (\rho_p \mathbf{V}_p) = 0,
\]

\[
\nabla \cdot \mathbf{V}_g = 0,
\]

\[
\frac{\partial \mathbf{V}_p}{\partial t} + \mathbf{V}_p \cdot \nabla \mathbf{V}_p = -\Omega_K^2 \mathbf{r} - \frac{\mathbf{V}_p - \mathbf{V}_g}{t_{\text{stop}}},
\]

\[
\frac{\partial \mathbf{V}_g}{\partial t} + \mathbf{V}_g \cdot \nabla \mathbf{V}_g = -\Omega_K^2 \mathbf{r} + \frac{\rho_p}{\rho_g} \frac{\mathbf{V}_p - \mathbf{V}_g}{t_{\text{stop}}} - \frac{\nabla P}{\rho_g}
\]
streaming instability

model system of drifting particles as an incompressible gas coupled aerodynamically to a compressible particle fluid

Youdin & Goodman (2005) showed this system is almost always linearly unstable.

Unstable modes and growth rate are $f(\tau, \eta, Z)$

- particle size
- density and temperature gradient in gas
- solid to gas ratio

[Graph showing contour lines and logarithmic scale]
new hypothesis for planetesimal formation

- Pairwise collisions grow solids to mm-cm scales
- Streaming instability **concentrates** particles until Roche density is exceeded

\[ \rho_R \sim \frac{M_*}{r^3} \]

- Self-gravity of the particles leads to collapse into planetesimals
simulations of collapse phase

- shearing box (local) in small domain: 0.2h x 0.2h x 0.2h
- isothermal, compressible, hydrodynamic gas (no MHD)
- solid fluid is represented by super-particles
- no explicit collisions
- Athena code
- self-gravity via Particle-Mesh scheme
\( \tau = 0.3, \ Z = 0.02 \)

512\(^3\) gas, 1.5 \times 10^8\) particles

nominal mass resolution corresponds to \(\sim 0.5\) km bodies

form \(\sim\) axisymmetric bands of clustered cm-sized solids

collapse gravitationally

Simon et al. 2016
• mass distribution of primordial bodies fit as a single power-law $dN/dm \sim m^{-1.6}$
• “top heavy” – most mass in largest planetesimals
do planetesimal properties depend on the size of the particles and/or the strength of the gas pressure gradient in the disk? 

hint from the model linear analysis

- **scale** of modes is directly proportional to $\eta$
- **mass** in associated volume proportional to $\eta^3$
- steep pressure gradients lead to bigger planetesimals?
zero pressure gradient

medium pressure gradient

strong pressure gradient

not unstable to streaming

Abod, Simon et al., in press
identify collapsed planetesimals

- fit as a single power-law
  - one parameter, the power-law index $p$

- fit as an exponentially tapered power-law
  - power-law index $p'$
  - cut-off mass $M_0$
no significant evidence that the power-law slope depends on the pressure gradient, except when $\eta = 0$

same result holds if we fix $\eta$ and vary $\tau$ (i.e. the particle size)
characteristic mass has a weak (possibly insignificant) scaling with $\eta$

$M_0 \sim \eta^3$ not consistent with numerical results

absolute scale of planetesimal masses is of the order of the mass given by the most unstable linear scale for a self-gravitating particle layer

$$M_G \propto \chi_G^2 \Sigma_p \propto \frac{\Sigma_p^3}{\Omega_K^4}$$
interpretation

• mass function of planetesimals formed from streaming initiated collapse is at least approximately “universal” – does not depend strongly on the properties of the solids or the gas disk

• mass function is derived from the non-linear clustering of solids in the turbulence excited by the instability – linear analysis of the model problem is a weak guide to the relevant scales

Simon et al. 2017
Abod, Simon et al., in press
faint Kuiper belt objects have an inferred size distribution that does not match the simulation prediction, but is not too far off…
Making progress toward modeling how the first large bodies in planetary systems formed...

Simulations: Jake Simon, Zhaohuan Zhu