Anisotropies in the Gravitational Wave Background from Cosmological Phase Transitions

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Gravitational Waves (GW)

Astrophysical sources

black hole, neutron star, white dwarf mergers

can be resolvable

study physics of gravity, QCD, galaxy formation,…
GW Cosmology

Cosmological sources

Phase transition (PT), inflation, pre-heating, cosmic string,…

unresolvable, stochastic

study Higgs sectors, Baryogenesis, inflation, reheating…
GW from first order Phase Transitions

Most discussions focus on GW energy/frequency spectrum from PT.

PT ~ TeV-100TeV => GW frequencies ~ proposed detectors!

studies on stochastic GW, see e.g. Romano & Cornish (2017)
GW from **first order PT**

However, the **anisotropic pattern** of GW provides valuable info on inflation/reheating.
GW anisotropies in other contexts

Astro sources: Cutler, Holz ’09; Cusin et al ’17
Inflationary preheating: Bethke et al ’13, ’14;
Analytic frameworks: Cusin et al ’17, Olmec et al ’12,
applied to Cosmic string networks: Jenkins et al ’18
Non-Gaussianities in pulsar timing arrays: Tsuneto et al ’19
Gravitational Wave Background (GWB)

Similar to the CMB spectrum, but with photon $\rightarrow$ GW from PT

hot spot $\rightarrow$

Higher energy photons

$\rightarrow$ Higher energy GW
Gravitational Wave Background (GWB)

Similar to the CMB spectrum, but with

$\text{photon} \rightarrow \text{GW from PT}$

where do hot / cold spots come from?
radius: red-shift of the CMB photon

later time

earlier time

constant temperature surface of last scattering
In a homogeneous universe

=> uniform photon redshift from last scattering
constant temperature surface of last scattering

redshift perturbation is of order $\sim 10^{-5}$
constant temperature surface of last scattering

redshift perturbation is of order $\sim 10^{-5}$

With primordial temperature fluctuations

$\Rightarrow$ anisotropic redshift for last scattering photons
With a single reheating process after inflation

=> GW fluctuations correlated with CMB
\[ C^{cross} \equiv \frac{\langle \rho_{GW}(1) \rho_{CMB}(2) \rangle}{\bar{\rho}_{GW} \bar{\rho}_{CMB}} \neq 0 \]

uniform critical temperature surface at PT

With only gravitational interactions

\[ \Rightarrow GW \text{ fluctuations nearly "pristine"} \]
Dark Ages of Cosmology

High scale inflation, B-modes?

$10^{14} \text{ GeV}(?)$  TeV  GeV  MeV  eV

BBN, LSS, CMB, BAO, …
GW can probe uncharted thermal history

Energy scale and physics of cosmological PT? - Frequency spectrum

Multiple sources of primordial density perturbations and reheating processes during/after inflation? - Anisotropies
If PT physics and CMB have different sources of primordial fluctuations and reheating history

=> GWB can be "uncorrelated" with CMB

\[
C_{cross} \equiv \frac{\langle \rho_{GW}(1)\rho_{CMB}(2) \rangle}{\bar{\rho}_{GW}\bar{\rho}_{CMB}} = 0
\]
Can we see the GW anisotropy?
GW from first order PT
First order phase transition

\[ V(\phi) \]

Temperature

Tunneling

\[ \Gamma(T) = A(T) e^{-S(T)} \]

PT rate as a function of temperature
GW from first order PT

- The dynamics and collisions of the bubbles generate gravity waves

In the sky today:

> $10^{30}$ bubbles from TeV scale PT

Cutting, Hindmarsh, Weir (2018)
Energy density of GW from PT

\[ \omega_{GW}^2 \delta g_{GW} \sim G_N \rho_{PT} \]
Energy density of GW from PT

Einstein eq.

\[ \omega^2_{GW} \delta g_{GW} \sim G_N \rho_{PT} \]

Typical frequency (micro-phys)

\[ \omega_{GW} \sim \frac{1}{\Delta t_{PT}} \sim \left(\frac{\dot{\Gamma}}{\Gamma}\right)_{T_{PT}} \]
Energy density of GW from PT

\[ \omega_{GW}^2 \delta g_{GW} \sim G_N \rho_{PT} \]

Typical frequency

\[ \omega_{GW} \sim \frac{1}{\Delta t_{PT}} \sim \left( \frac{\dot{\Gamma}}{\Gamma} \right)_{T_{PT}} \]

Energy density in GW

\[ \rho_{GW} \sim \frac{1}{G_N} \omega_{GW}^2 \left( \delta g_{GW} \right)^2 \]
Energy density of GW from PT

Einstein eq. $\omega_{GW}^2 \delta g_{GW} \sim G_N \rho_{PT}$

Typical frequency $\omega_{GW} \sim \frac{1}{\Delta t_{PT}} \sim \left(\frac{\dot{\Gamma}}{\Gamma}\right)_{T_{PT}}$

Energy density in GW $\rho_{GW} \sim \frac{1}{G_N} \omega_{GW}^2 (\delta g_{GW})^2$

$H_{PT}^2 \sim G_N \rho_{total}$

$\rho_{GW} \sim \frac{\rho_{PT}^2}{\rho_{total}} (H_{PT} \Delta t_{PT})^2$
Energy density of GW from PT

\[ \rho_{GW} \sim \frac{\rho_{PT}^2}{\rho_{total}} (H_{PT} \Delta t_{PT})^2 \]

Typical Estimate: \( H_{PT} \Delta t_{PT} \sim 10^{-2} \)

RS1/Composite Higgs

Hawking-Page/conformal -> confinement PT: \( H_{PT} \Delta t_{PT} \rightarrow 1 \)

Randall, Servant '07; Konstandin, Servant '11

(or ordinary PT by tuning)

See 1512.06239 for a review of PT models

\[ h^2 \Omega_{env}(f) = 1.67 \times 10^{-5} \left( \frac{H_*}{\beta} \right)^2 \left( \frac{\kappa \alpha}{2b + \alpha} \right)^2 \left( \frac{100}{g_*} \right)^{\frac{1}{3}} \left( \frac{0.11 v_w^3}{0.42 + v_w^2} \right) S_{env}(f) \]
GW from PT

\[ \rho_{GW} \sim \frac{\rho_{PT}^2}{\rho_{total}} (H_{PT} \Delta t_{PT})^2 \]

\[ \rho_{GW}^{today} \approx 0.1 (H_{PT} \Delta t_{PT})^2 \rho_\gamma \approx 10^{-5} - 10^{-2} \rho_\gamma \]

< CMB N_{eff} bounds

\[ \omega_{GW}^{today} \sim \omega_{GW} \left( \frac{T_{CMB}^{today}}{T_{PT}} \right) \gtrsim \text{mHz - Hz}. \]
GW detectors

Laser Interferometer Space Antenna

2.5 \times 10^5 \text{ km}

Similar idea, more satellites, more futuristic

BBO, DECIGO, ALIA

plus atomic interferometry: MAGIS
Energy density of GW from PT

\[ \Omega_{GW} h^2 \]

Frequency (Hz)

PT (isotropic)

LIGO

GW150914

eLISA

LISA

Unresolvable galactic binaries

ALIA

DECIGO

BBO

rholoc.com/apps/GWplotter/
Seeing the anisotropic pattern
Angular measurement

**Method:** variation of strains in time for each polarization mode with different detector location / doppler shift
A deconvolution problem

\[ \tilde{C}(t, f) = \frac{1}{4\pi} \sum_{\ell m} \left[ p_{\ell m}^E(f) \right]^* a_{\ell m}^E(f, t) \]

output data  GW background  antenna pattern
Using cross correlation between different phase readout, **LISA** may get to $\ell_{\text{max}} \sim 10$, more detectors (**BBO** / **DECIGO**) can do much better [e.g., Cutler & Holz (2009)]
Astrophysical foreground

Unresolvable white dwarf merger generates the dominant background to our signal

However, most of these backgrounds follow galaxy distribution and can be subtracted with enough data

Adams & Cornish (2013)  
Anisotropic GW: minimal story
Anisotropic Signal

- Natural to have anisotropic GW signal (like CMB)

\[ \rho_{GW}(\theta, \phi) = \bar{\rho}_{GW} + \delta \rho_{GW}(\theta, \phi) \]

- Two-point correlators of signal fluctuations

\begin{align*}
\text{GW-GW} & \quad C^{GW}(\theta) \equiv \langle \rho_{GW}(1) \rho_{GW}(2) \rangle_{\theta} \frac{1}{\bar{\rho}_{GW}^2} \\
\text{GW-CMB} & \quad C^{\text{cross}}(\theta) \equiv \langle \rho_{GW}(1) \rho_{\text{CMB}}(2) \rangle_{\theta} \frac{1}{\bar{\rho}_{GW} \bar{\rho}_{\text{CMB}}} 
\end{align*}
In order to see the anisotropy

\[ C^{GW}(\hat{n}) = \sum_{\ell m} C_{\ell m} Y_{\ell m}(\hat{n}) \]

\[ \sqrt{C_{\ell}} > \text{detector sensitivity} \]

Angular resolution \(< \pi \ell_{\text{max}}^{-1} \)

Baumann, TASI lecture
Minimal Story

• Single source of primordial perturbations (= quantum fluctuations in inflaton field)

• GW anisotropy is totally correlated with primordial photon perturbation

• Roughly scale-invariant primordial perturbation:

\[
\left( \frac{\delta \rho_{GW}}{\rho_{GW}} \right)_\ell \sim \sqrt{C_{\ell}^{GW}} \sim \frac{10^{-5}}{\ell}
\]
Detection possibility

\[ \delta \rho_{GW}^{today} \approx 10^{-10} - 10^{-7} \rho_{\gamma} \]
Detection possibility

$$\delta \rho^{today}_{GW} \approx 10^{-10} - 10^{-7} \rho_\gamma$$
Detection possibility

\[ \delta \rho_{GW}^{today} \approx 10^{-10} - 10^{-7} \rho_\gamma \]

PT (isotropic) may see the anisotropy in BBO up to \( l \approx 100 \).
A Non-minimal Story
Non-minimal story

- There could be multiple sources of primordial fluctuations
- The GW and CMB maps are not necessarily correlated
In addition to the inflaton there is an Axion-Like Particle with quantum fluctuations during inflation

\[ V = \Lambda^4 \left( 1 - \sin \frac{a}{f_a} \right) \]

\[ \frac{\delta \rho_a}{\rho_a} \sim \frac{\delta V}{V} \sim \frac{H_{\text{inf}}}{f_a} \]

can generate (possibly larger!) uncorrelated perturbations to the inflaton fluctuations
e.g. a curvaton model

ALP with smaller energy density but larger perturbation decays into visible sector (VS) particles, while inflaton decays into a hidden sector (HS)

\[
\left( \frac{\delta \rho}{\rho} \right)_{\text{HS}} \leq 10^{-5}
\]

\[
\left( \frac{\delta \rho}{\rho} \right)_{\text{VS}} \geq 10^{-5}
\]

remain uncorrelated if HS-VS are mostly decoupled
**e.g. a curvaton model**

**VS** undergoes a strong first order PT, producing GW with VS perturbation

![Diagram showing the phase transition](image)

\[
\left( \frac{\delta \rho}{\rho} \right)_{HS} \leq 10^{-5}
\]

\[
\left( \frac{\delta \rho}{\rho} \right)_{GW} \sim \left( \frac{\delta \rho}{\rho} \right)_{VS} \geq 10^{-5}
\]
e.g. a curvaton model

**HS** decays into **VS**, dominates energy density and suppresses photon perturbation to the observed value

\[
\left( \frac{\delta \rho}{\rho} \right)_{\text{CMB}} \sim \left( \frac{\rho_{\text{VS}}}{\rho_{\text{HS}}} \right) \left( \frac{\delta \rho}{\rho} \right)_{\text{GW}} + \left( \frac{\delta \rho}{\rho} \right)_{\text{HS}} \\
\sim 10^{-5}
\]
Correlated GWB & CMB

If density perturbation is dominated by the 1st term

\[
\left( \frac{\delta \rho}{\rho} \right)_{\text{CMB}} \sim \left( \frac{\rho_{\text{VS}}}{\rho_{\text{HS}}} \right) \left( \frac{\delta \rho}{\rho} \right)_{\text{GW}} + \left( \frac{\delta \rho}{\rho} \right)_{\text{HS}} \sim 10^{-5}
\]

the CMB and GW background are completely correlated

\[
C^{\text{cross}} \equiv \frac{\langle \rho_{\text{GW}}(1) \rho_{\text{CMB}}(2) \rangle}{\bar{\rho}_{\text{GW}} \bar{\rho}_{\text{CMB}}} \neq 0
\]
Un-correlated GWB & CMB

If density perturbation is dominated by the 2nd term

\[
\left(\frac{\delta \rho}{\rho}\right)_{CMB} \sim \left(\frac{\rho_{VS}}{\rho_{HS}}\right) \left(\frac{\delta \rho}{\rho}\right)_{GW} + \left(\frac{\delta \rho}{\rho}\right)_{HS} \sim 10^{-5}
\]

the CMB and GW background are completely uncorrelated

\[
C_{cross} \equiv \frac{\langle \rho_{GW}(1) \rho_{CMB}(2) \rangle}{\bar{\rho}_{GW} \bar{\rho}_{CMB}} = 0
\]
e.g. a curvaton model

\[ \delta \rho_{GW} \sim 0.1 \left( \frac{\rho_{VS}}{\rho_{HS}} \right)^2 (H_{PT} \Delta t_{PT})^2 \left( \frac{\delta \rho}{\rho} \right)_{GW} \rho_\gamma < \text{CMB bound on isocurvature} \]

\[ H_{PT} \Delta t_{PT} = 0.1 \]

\[ \left( \frac{\delta \rho}{\rho} \right)_{GW} = 10^{-4} \quad \frac{\rho_{VS}}{\rho_{HS}} = 0.1 \]

larger energy contrast

anisotropy is visible at BBO up to \( \ell_{\text{max}} \approx 100 \)
Conclusion and Outlook

First Order PT Re-examine RS1

Gravitational Waves

Inflaton/Curvaton Perturbations

Anisotropic GW Background

Future detectors

Non-Gaussianity?