

# Anisotropies in the Gravitational Wave

Background from Cosmological Phase Transitions

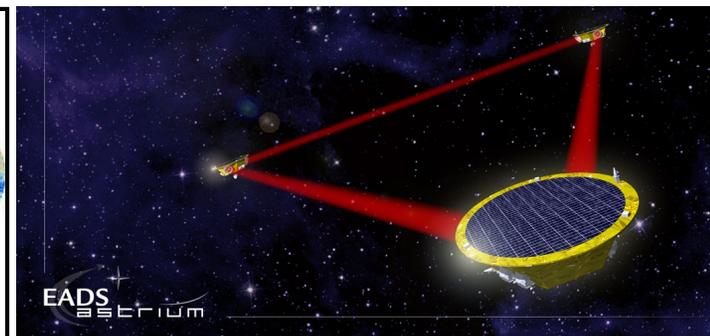
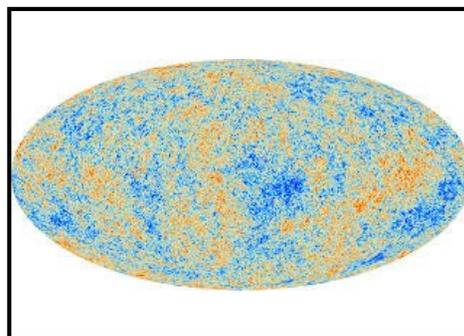
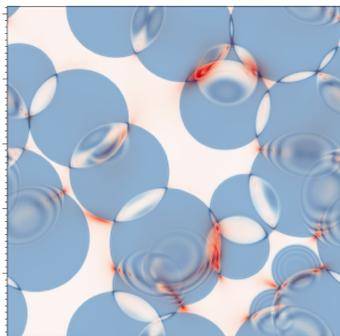
Raman Sundrum

(borrowing from Yuhsin Tsai)

University of Maryland

PRL 121, 201303 (2018), arXiv:1803.10780

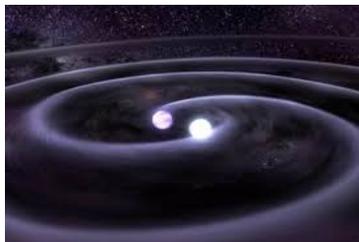
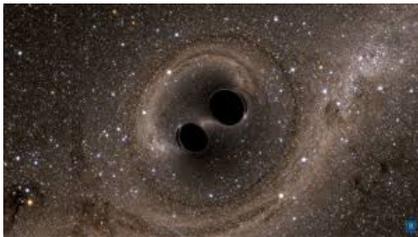
Michael Geller, Anson Hook, Raman Sundrum, Yuhsin Tsai



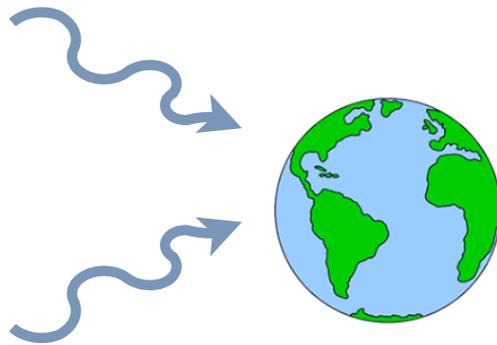
# Gravitational Waves (GW)

## Astrophysical sources

black hole, neutron star, white dwarf mergers



can be resolvable

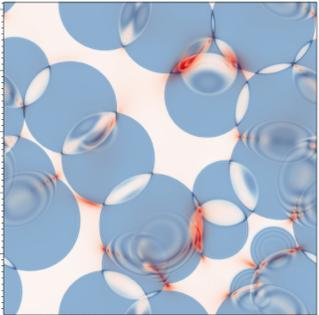


study physics of  
gravity, QCD,  
galaxy formation,...

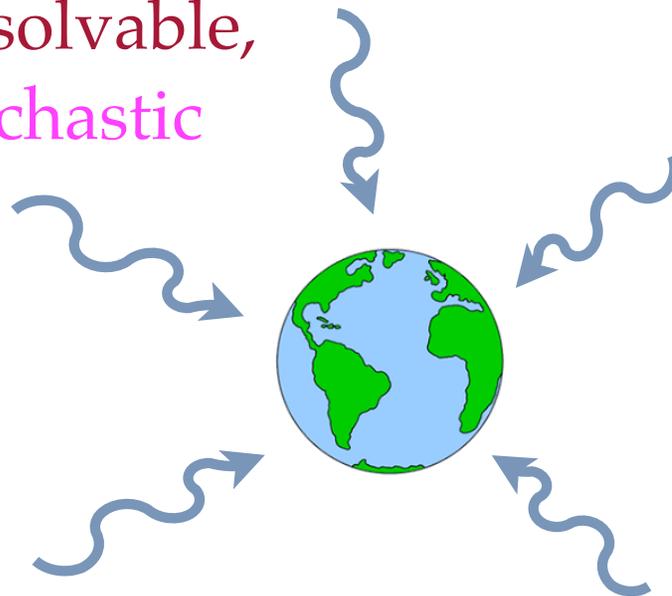
# GW Cosmology

## Cosmological sources

Phase transition (PT), inflation, pre-heating, cosmic string,...



unresolvable,  
stochastic

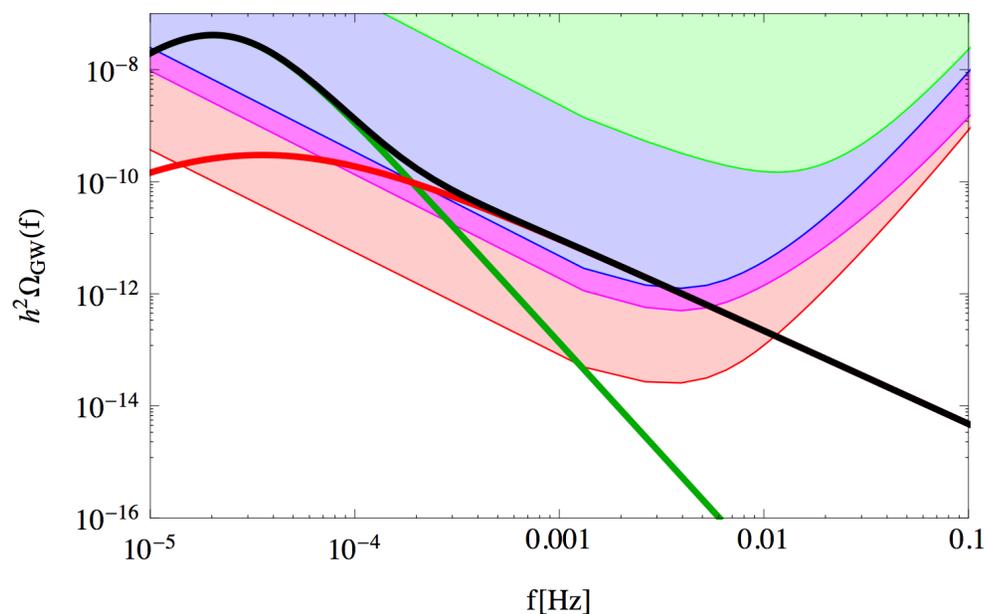
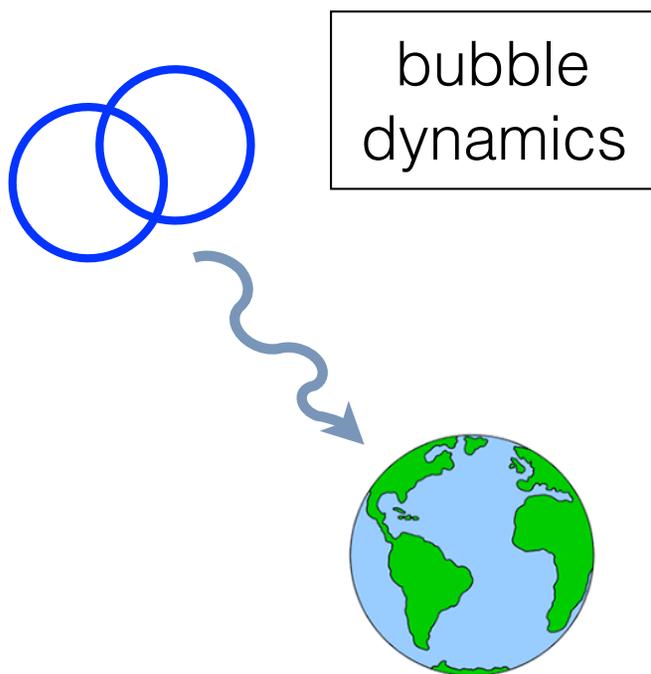


study Higgs sectors,  
Baryogenesis,  
inflation, reheating...

# GW from first order Phase Transitions

Most discussions focus on  
GW energy / frequency spectrum from PT.

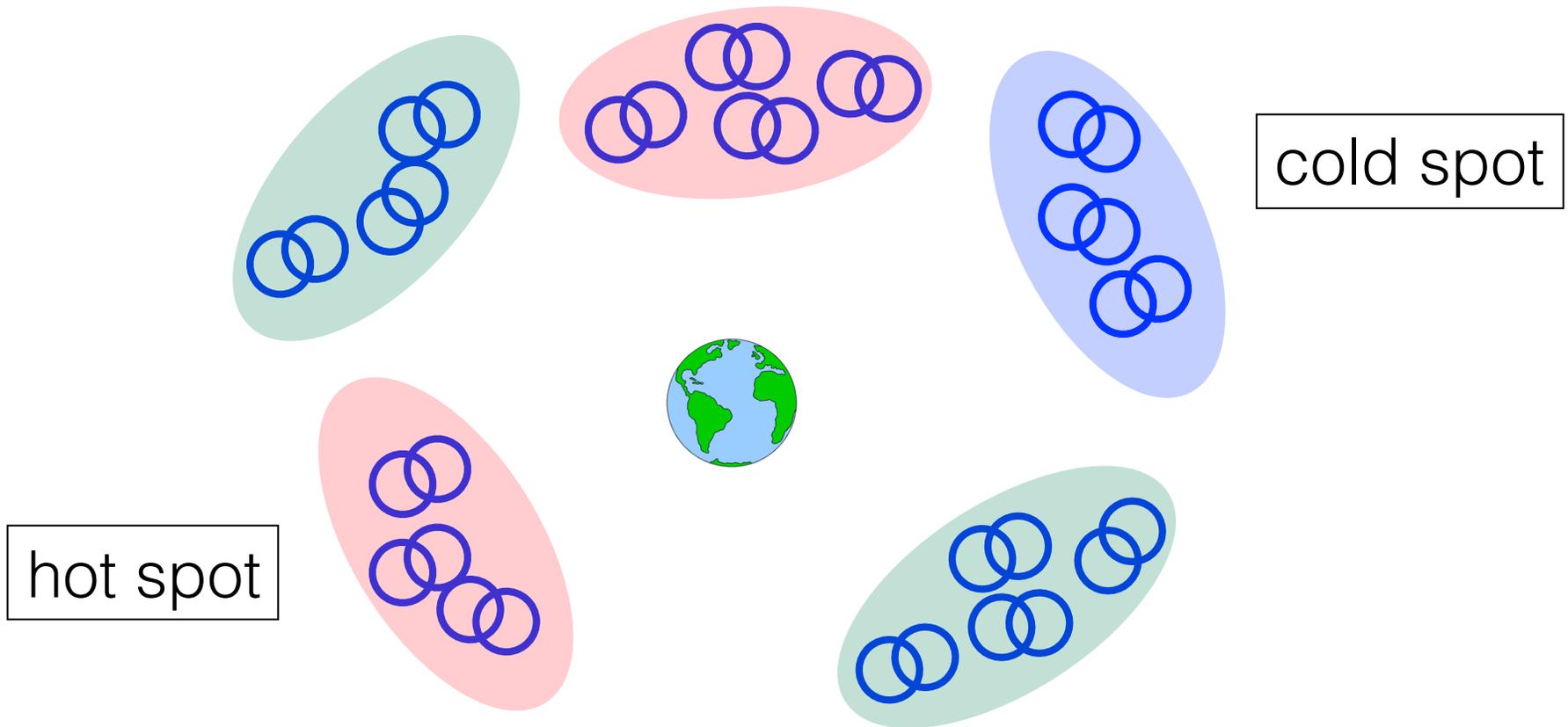
PT  $\sim$  TeV-100TeV  $\Rightarrow$  GW frequencies  $\sim$  proposed detectors!



studies on stochastic GW, see e.g. Romano & Cornish (2017) <sup>1512.06239</sup>

# GW from first order PT

However, the **anisotropic pattern** of GW provides valuable info on inflation/reheating



# GW anisotropies in other contexts

Astro sources: Cutler, Holz '09; Cusin et al '17

Inflationary preheating: Bethke et al '13, '14;

Analytic frameworks: Cusin et al '17, Olmec et al '12,

applied to Cosmic string networks: Jenkins et al '18

Non-Gaussianities in pulsar timing arrays: Tsuneto et al '19

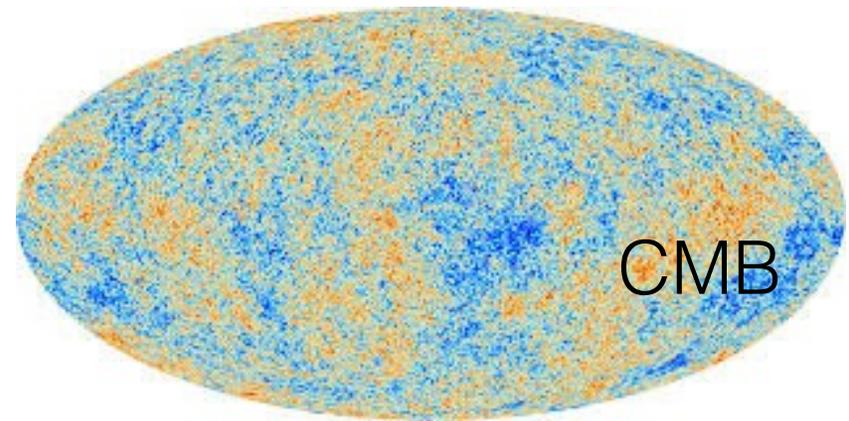
# Gravitational Wave Background (GWB)

Similar to the CMB spectrum, but with

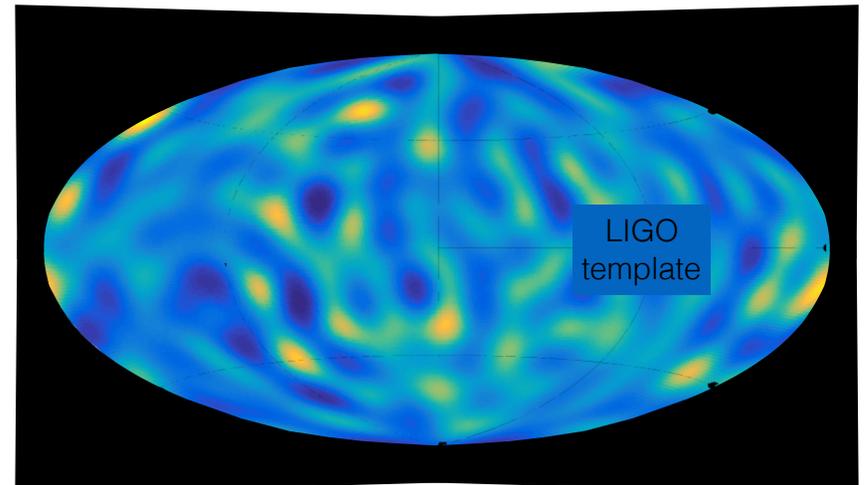
photon  $\rightarrow$  GW from PT

hot spot  $\Rightarrow$

Higher energy photons

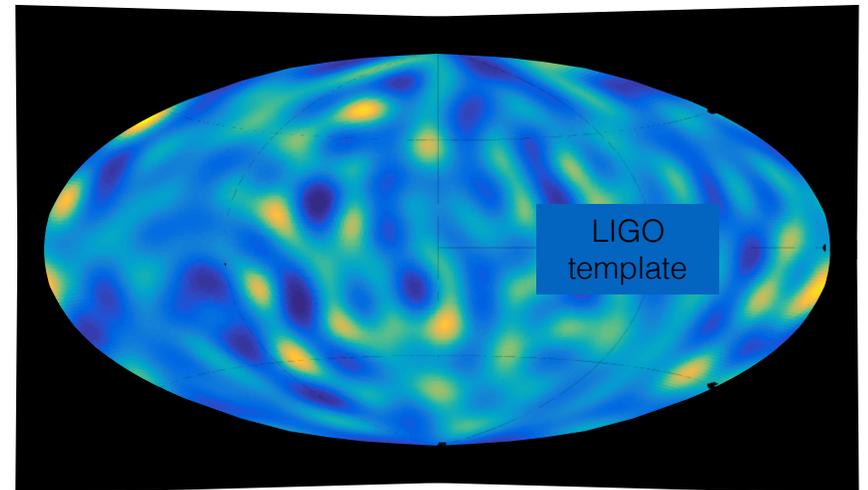
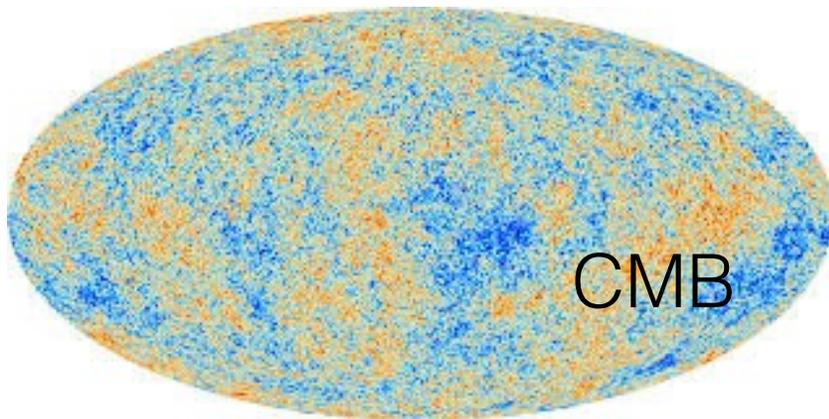


$\rightarrow$  Higher energy GW



# Gravitational Wave Background (GWB)

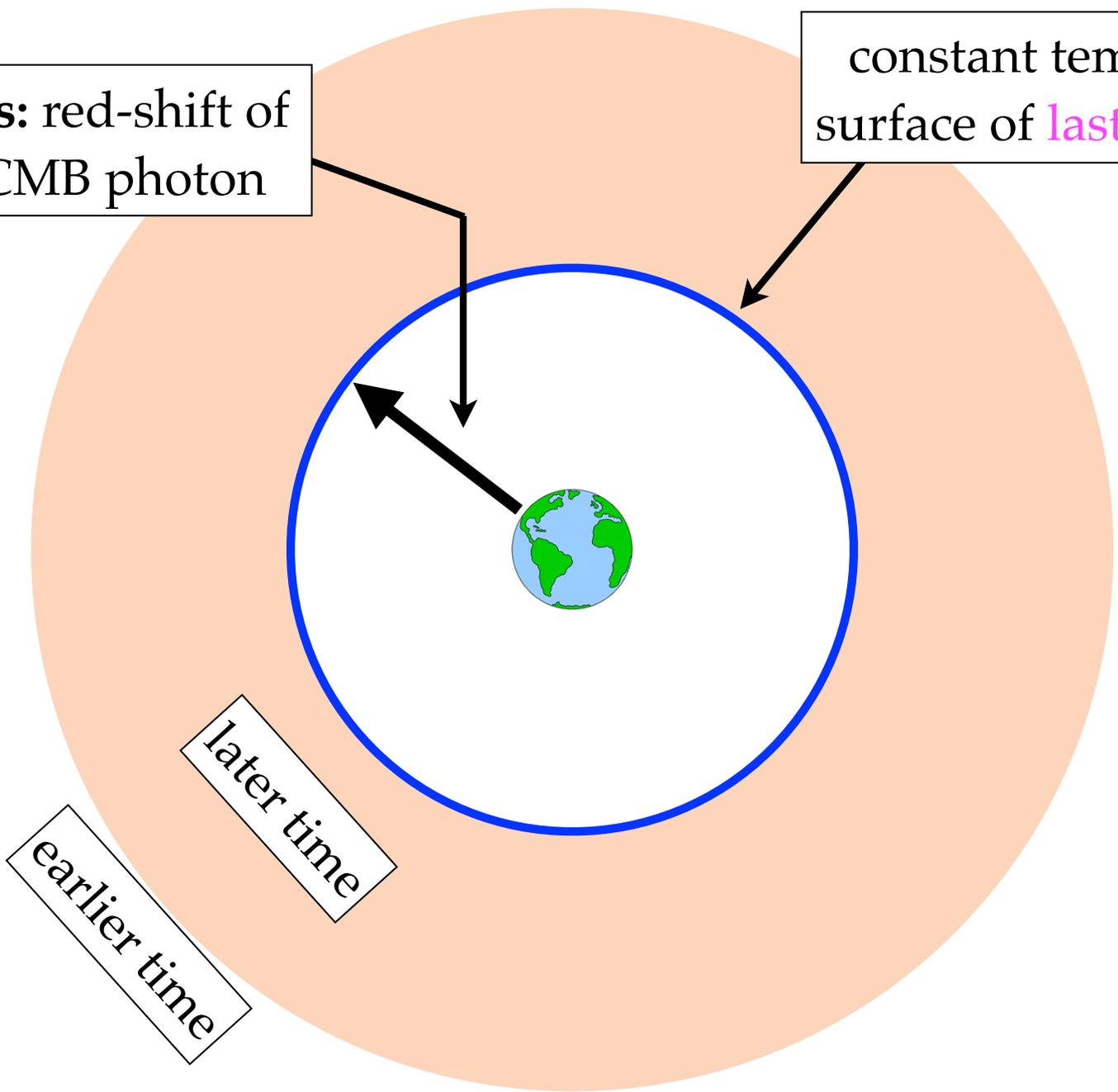
Similar to the CMB spectrum, but with  
photon  $\rightarrow$  GW from PT



where do hot / cold spots come from?

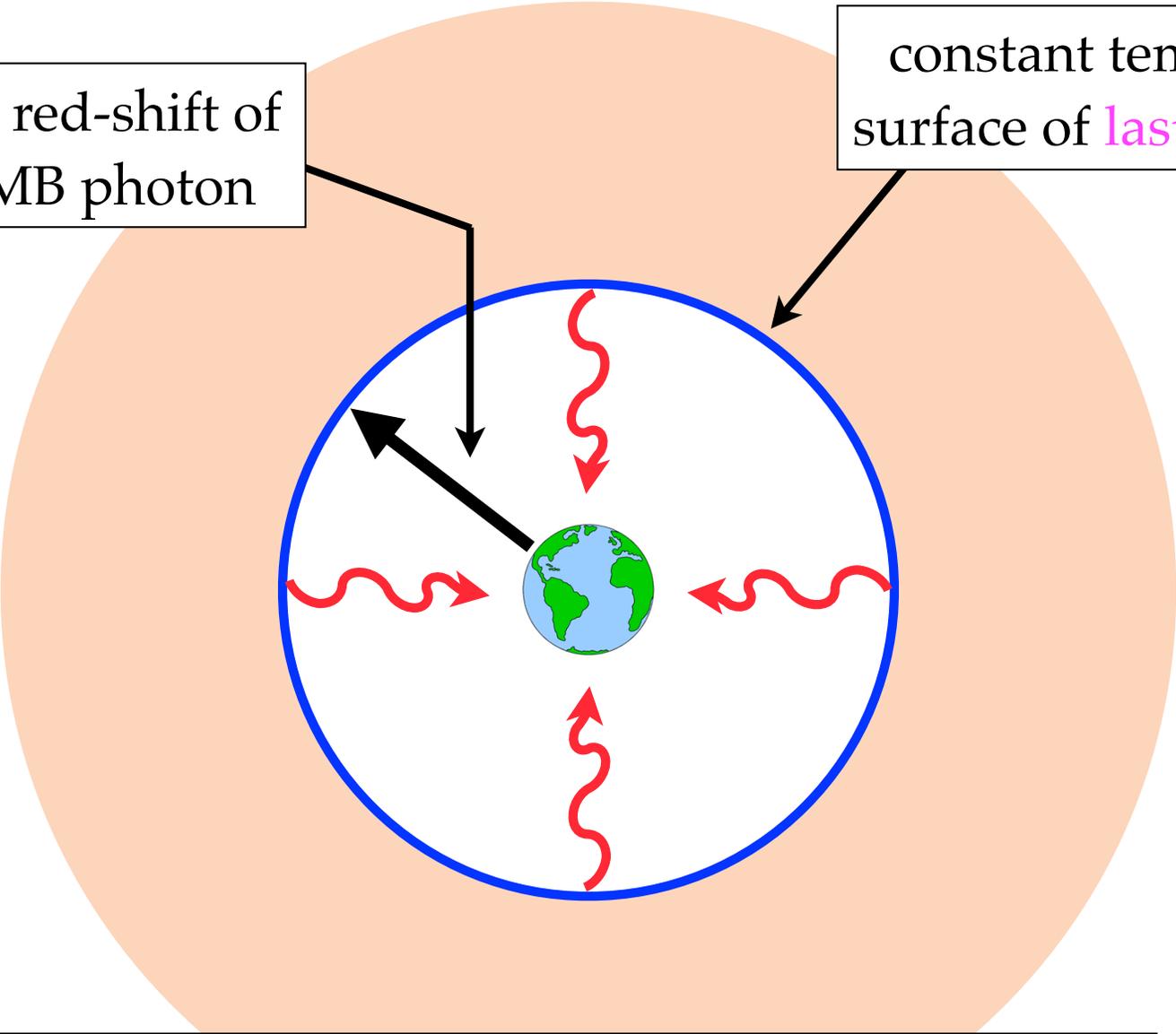
**radius:** red-shift of the CMB photon

constant temperature surface of **last scattering**

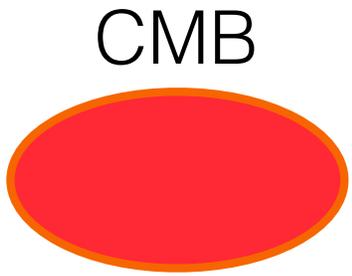


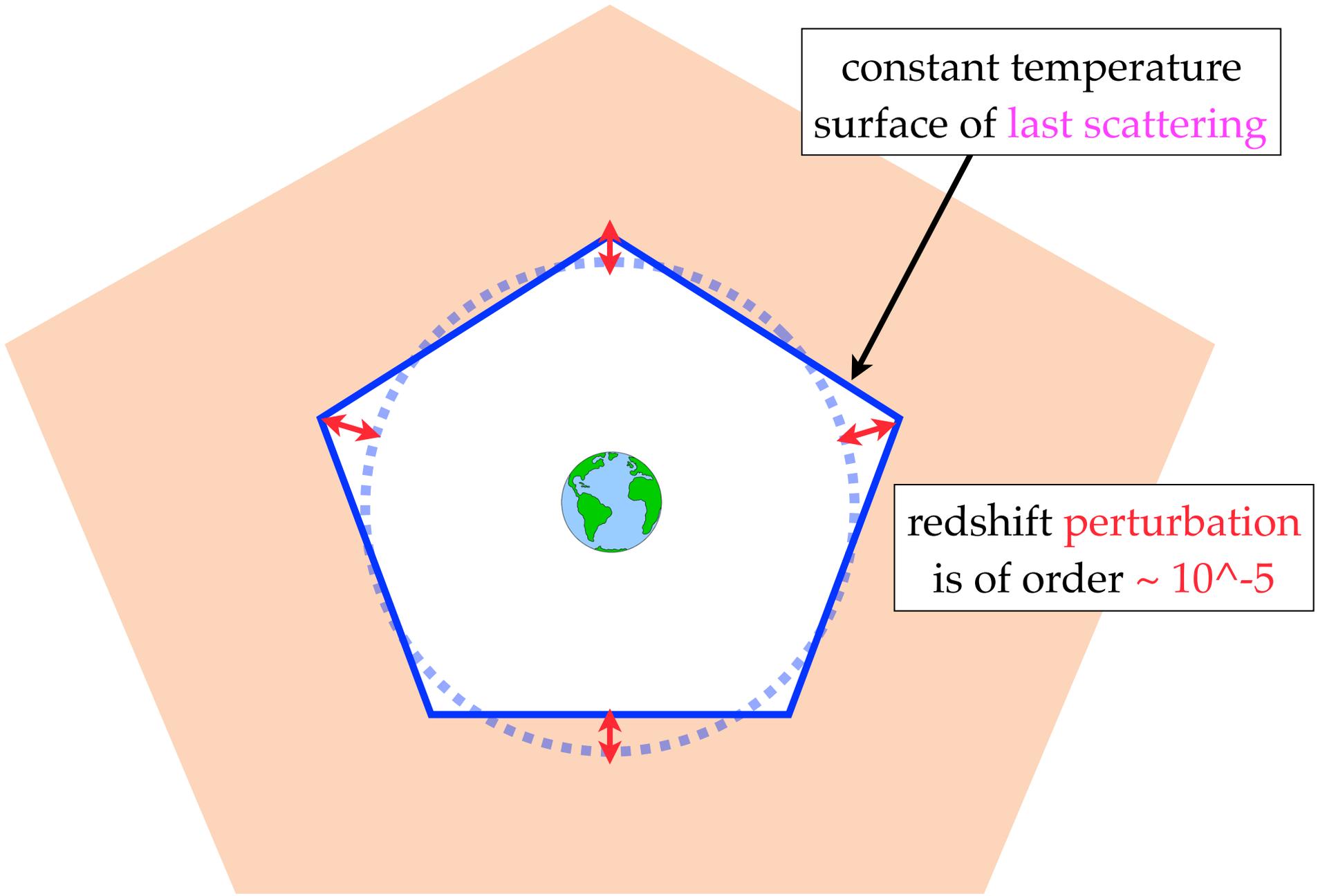
radius: red-shift of the CMB photon

constant temperature surface of last scattering



In a homogeneous universe  
=> uniform photon redshift from last scattering



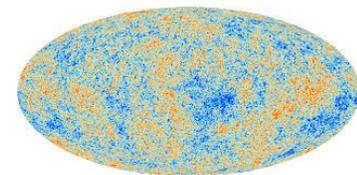


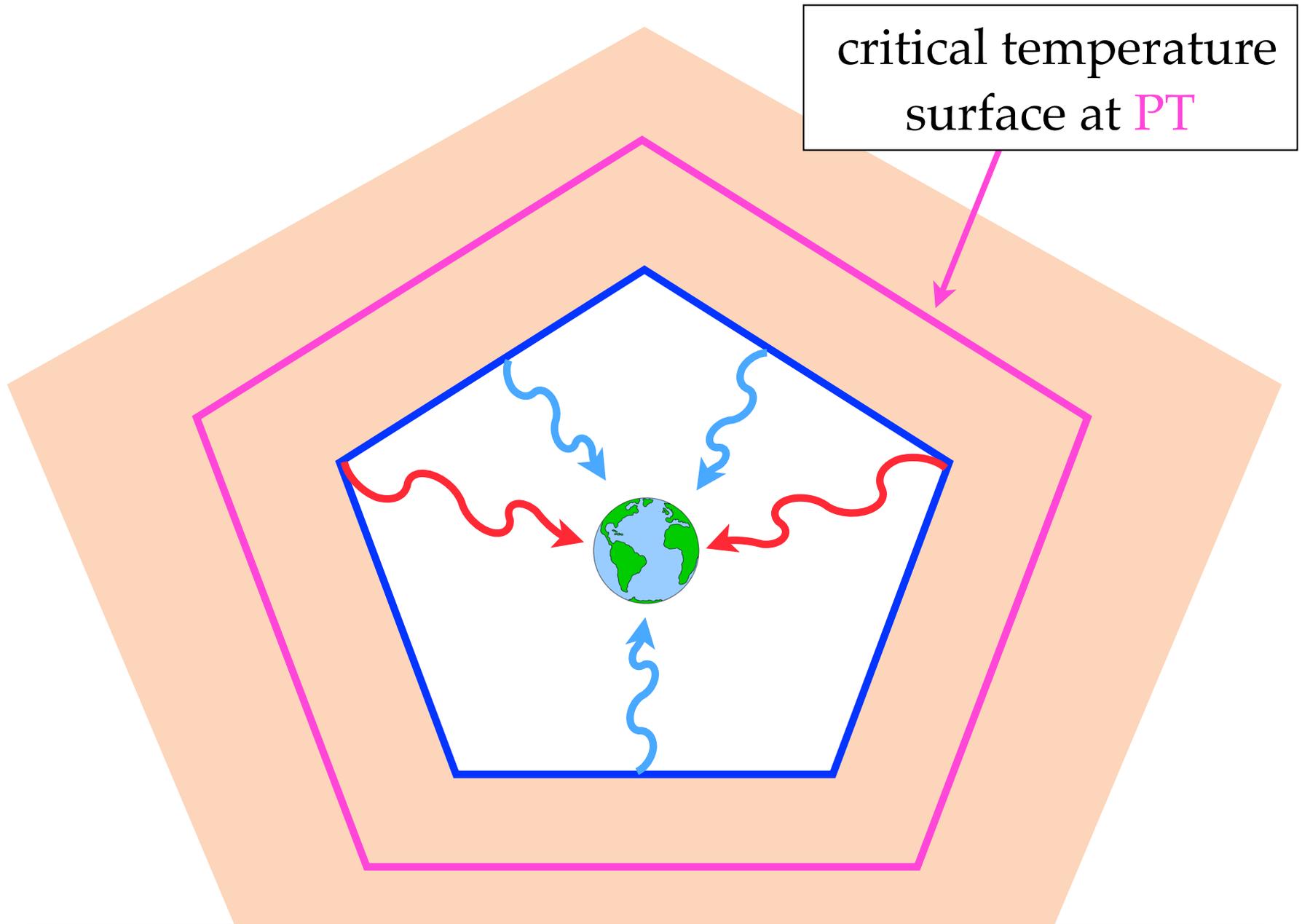
constant temperature  
surface of **last scattering**

redshift **perturbation**  
is of order  $\sim 10^{-5}$

With **primordial temperature fluctuations**  
 $\Rightarrow$  anisotropic redshift for last scattering photons

CMB

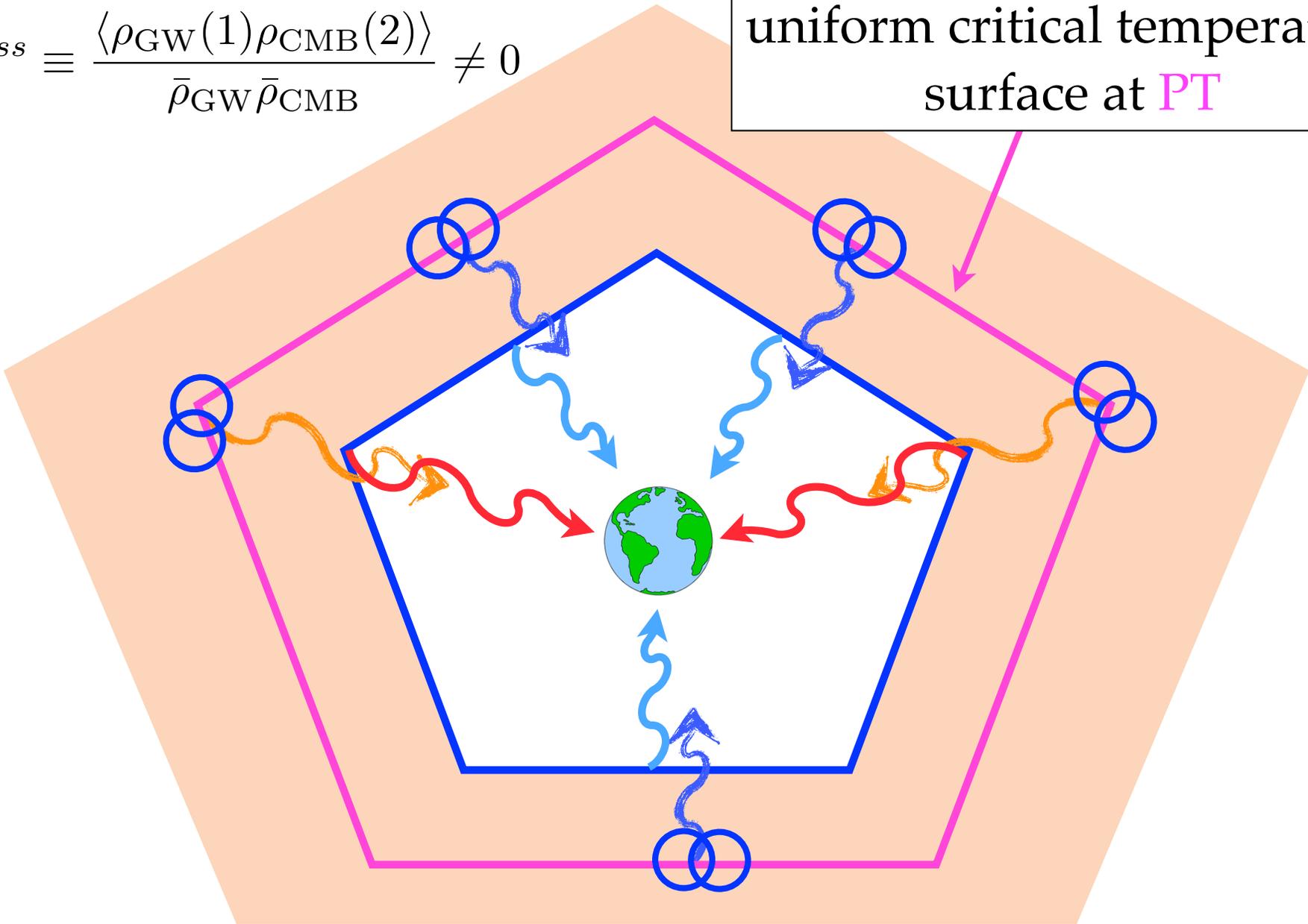




With a single reheating process after inflation  
=> GW fluctuations correlated with CMB

$$C^{cross} \equiv \frac{\langle \rho_{GW}(1) \rho_{CMB}(2) \rangle}{\bar{\rho}_{GW} \bar{\rho}_{CMB}} \neq 0$$

uniform critical temperature surface at PT



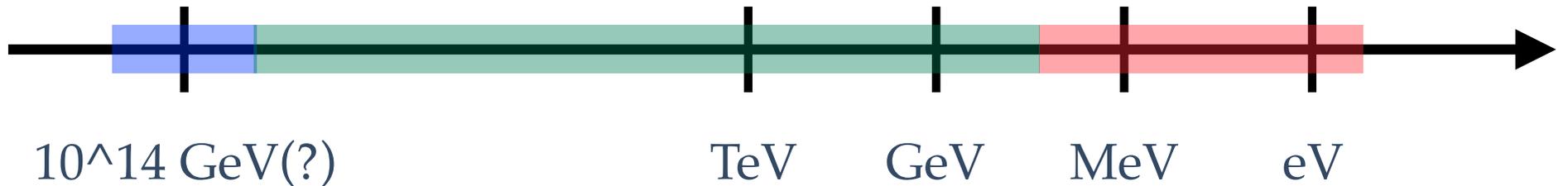
With only gravitational interactions  
 => GW fluctuations nearly "pristine"

# Dark Ages of Cosmology

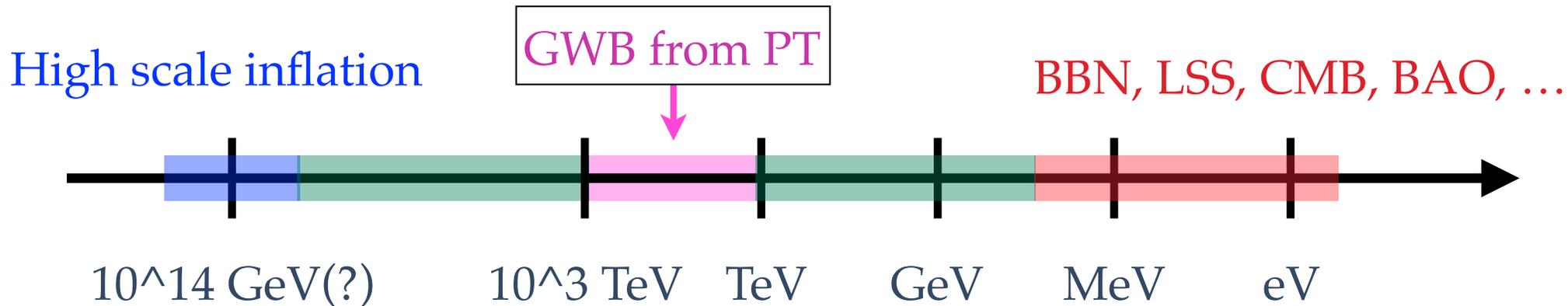
High scale inflation,  
B-modes?

?

BBN, LSS, CMB, BAO, ...



# GW can probe uncharted thermal history

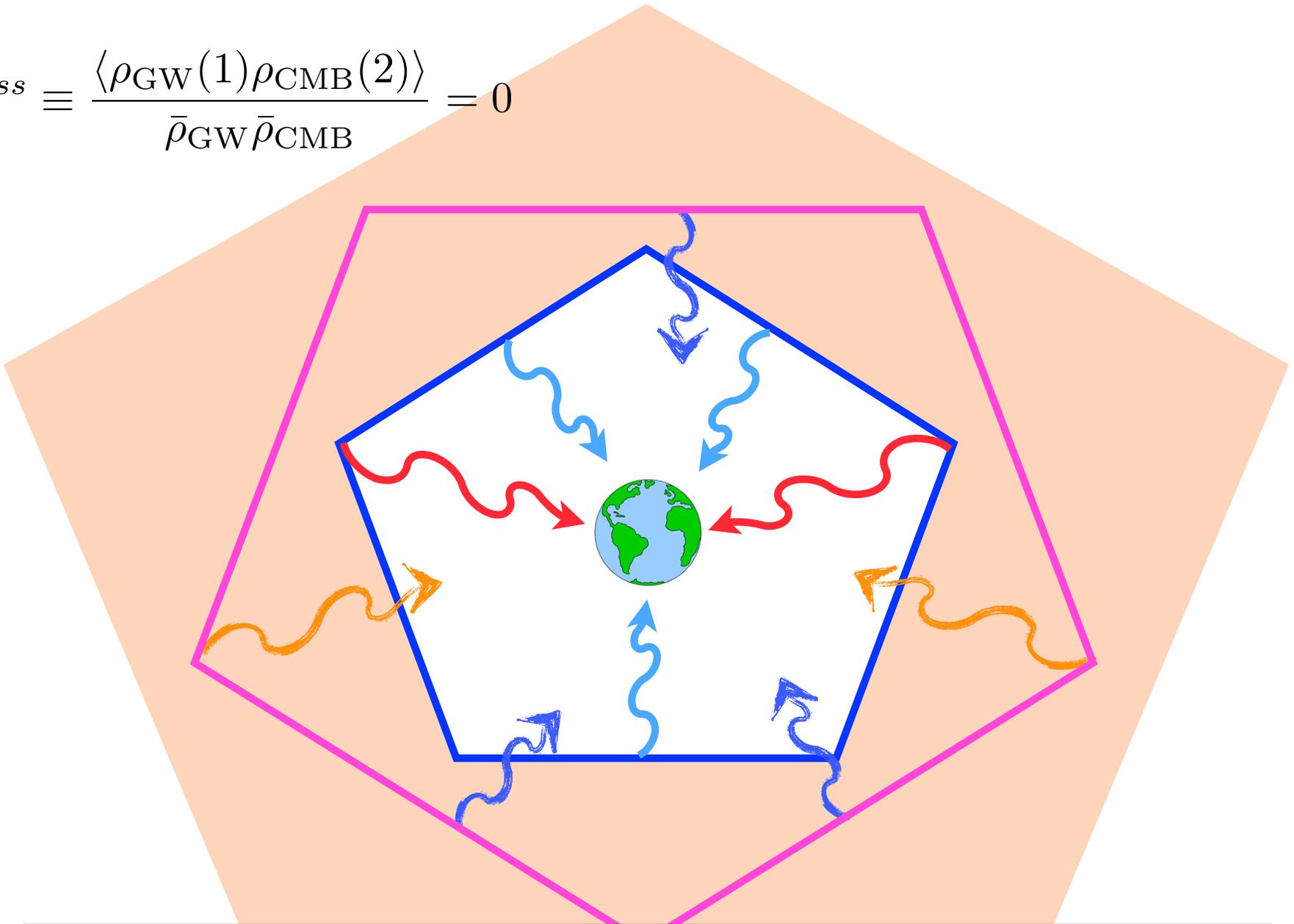


Energy scale and physics of cosmological PT? - Frequency spectrum

Multiple sources of primordial density perturbations and reheating processes during / after inflation? - Anisotropies

Eg.

$$C^{cross} \equiv \frac{\langle \rho_{GW}(1) \rho_{CMB}(2) \rangle}{\bar{\rho}_{GW} \bar{\rho}_{CMB}} = 0$$

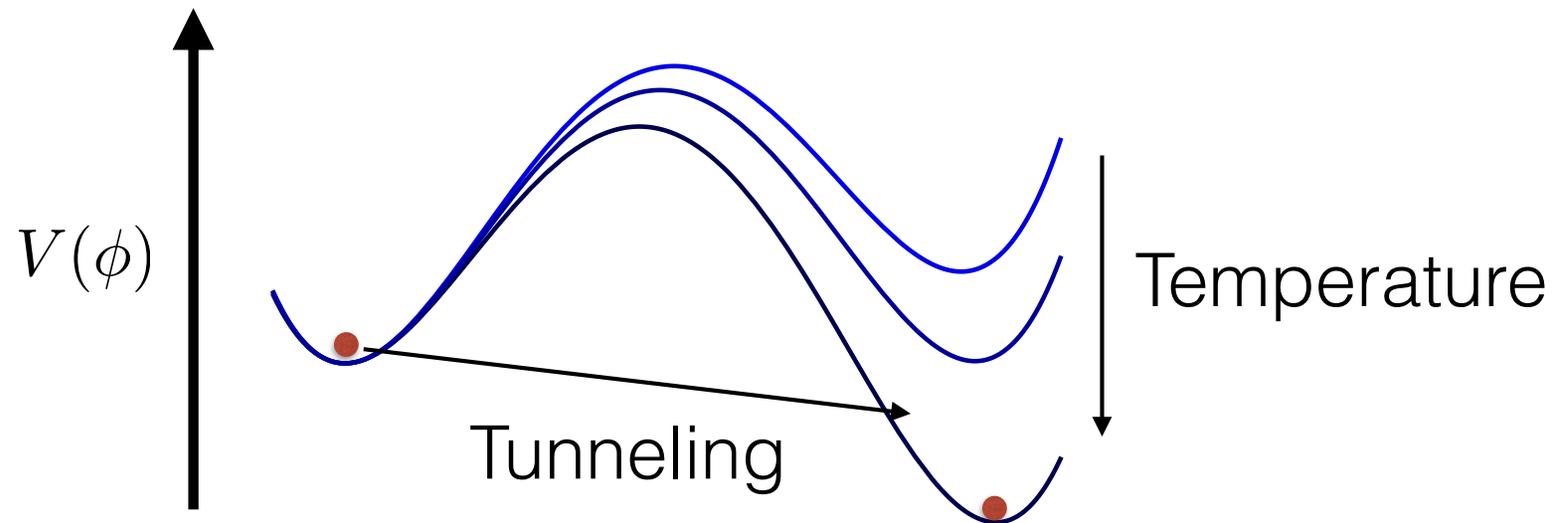


If PT physics and CMB have different sources of primordial fluctuations and reheating history  
=> GWB can be “uncorrelated” with CMB

Can we see the GW anisotropy?

GW from first order PT

# First order phase transition

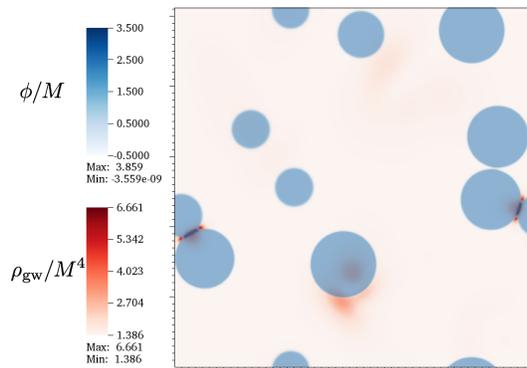


$$\Gamma(T) = A(T) e^{-S(T)}$$

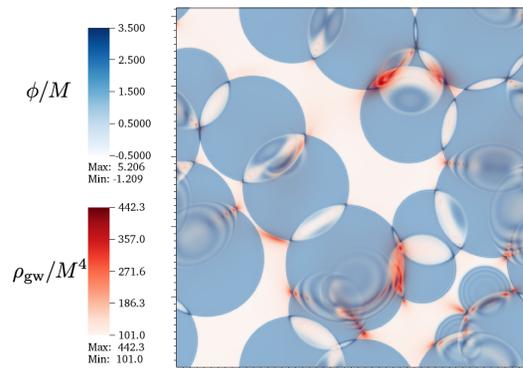
PT rate as a function of temperature

# GW from first order PT

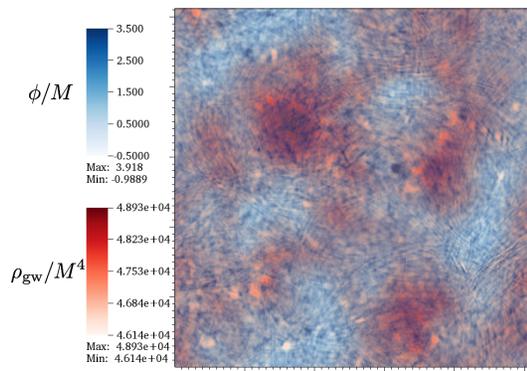
- The dynamics and collisions of the bubbles generate gravity waves



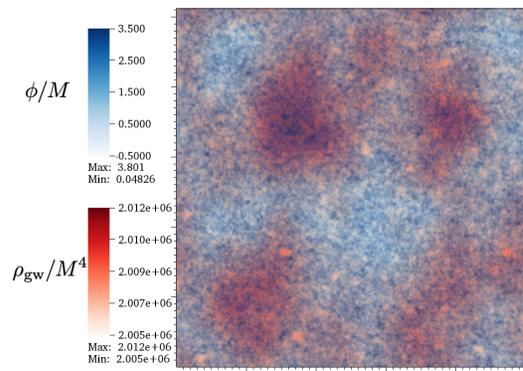
(a)  $t/R_* = 0.35$



(b)  $t/R_* = 0.66$



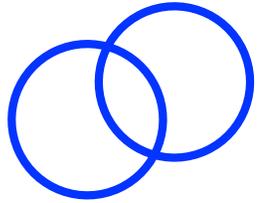
(c)  $t/R_* = 2.50$



(d)  $t/R_* = 7.8$

In the sky today:  
 $> 10^{30}$  bubbles  
from TeV scale PT

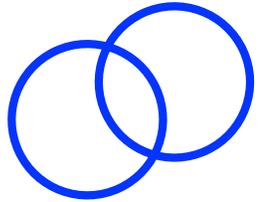
# Energy density of GW from PT



Einstein eq.

$$\omega_{\text{GW}}^2 \delta g_{\text{GW}} \sim G_N \rho_{\text{PT}}$$

# Energy density of GW from PT



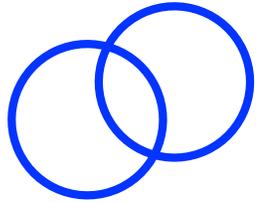
Einstein eq.

$$\omega_{\text{GW}}^2 \delta g_{\text{GW}} \sim G_N \rho_{PT}$$

Typical frequency  
(micro-phys)

$$\omega_{\text{GW}} \sim \frac{1}{\Delta t_{PT}} \sim \left( \frac{\dot{\Gamma}}{\Gamma} \right)_{T_{PT}}$$

# Energy density of GW from PT



Einstein eq.

$$\omega_{\text{GW}}^2 \delta g_{\text{GW}} \sim G_N \rho_{\text{PT}}$$

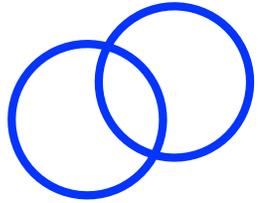
Typical frequency

$$\omega_{\text{GW}} \sim \frac{1}{\Delta t_{\text{PT}}} \sim \left( \frac{\dot{\Gamma}}{\Gamma} \right)_{T_{\text{PT}}}$$

Energy density in GW

$$\rho_{\text{GW}} \sim \frac{1}{G_N} \omega_{\text{GW}}^2 (\delta g_{\text{GW}})^2$$

# Energy density of GW from PT



Einstein eq.

$$\omega_{\text{GW}}^2 \delta g_{\text{GW}} \sim G_N \rho_{\text{PT}}$$

Typical frequency

$$\omega_{\text{GW}} \sim \frac{1}{\Delta t_{\text{PT}}} \sim \left( \frac{\dot{\Gamma}}{\Gamma} \right)_{T_{\text{PT}}}$$

Energy density in GW

$$\rho_{\text{GW}} \sim \frac{1}{G_N} \omega_{\text{GW}}^2 (\delta g_{\text{GW}})^2$$

$$H_{\text{PT}}^2 \sim G_N \rho_{\text{total}}$$

$$\rho_{\text{GW}} \sim \frac{\rho_{\text{PT}}^2}{\rho_{\text{total}}} (H_{\text{PT}} \Delta t_{\text{PT}})^2$$

# Energy density of GW from PT

$$\rho_{\text{GW}} \sim \frac{\rho_{PT}^2}{\rho_{\text{total}}} (H_{PT} \Delta t_{PT})^2$$

Typical Estimate:

$$H_{PT} \Delta t_{PT} \sim 10^{-2}$$

RS1 / Composite Higgs

Hawking-Page / conformal  $\rightarrow$  confinement PT:  $H_{PT} \Delta t_{PT} \rightarrow 1$

Randall, Servant '07; Konstandin, Servant '11

(or ordinary PT by tuning)

See 1512.06239 for a review of PT models

$$h^2 \Omega_{\text{env}}(f) = 1.67 \times 10^{-5} \left( \frac{H_*}{\beta} \right)^2 \left( \frac{\kappa \alpha}{2\epsilon + \alpha} \right)^2 \left( \frac{100}{g_*} \right)^{\frac{1}{3}} \left( \frac{0.11 v_w^3}{0.42 + v_w^2} \right) S_{\text{env}}(f)$$

# GW from PT

$$\rho_{\text{GW}} \sim \frac{\rho_{PT}^2}{\rho_{\text{total}}} (H_{PT} \Delta t_{PT})^2$$

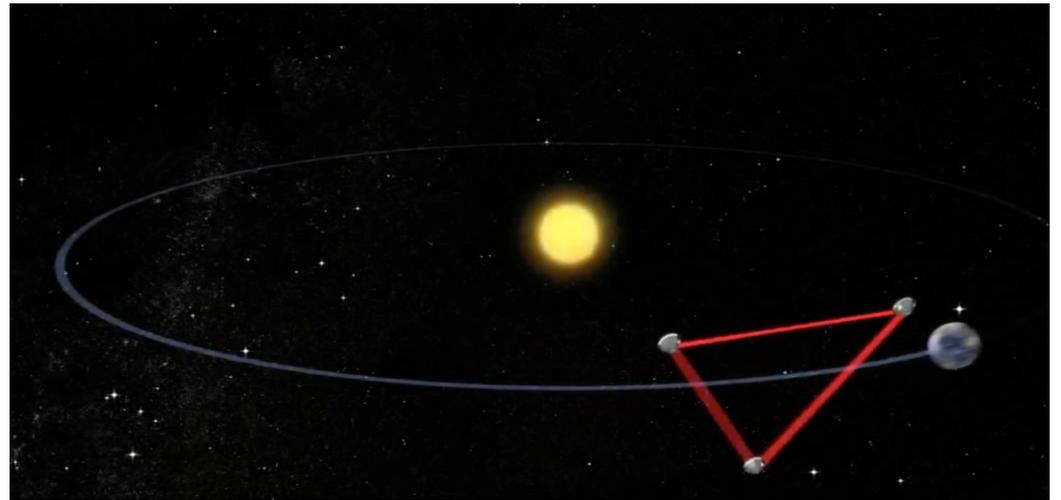
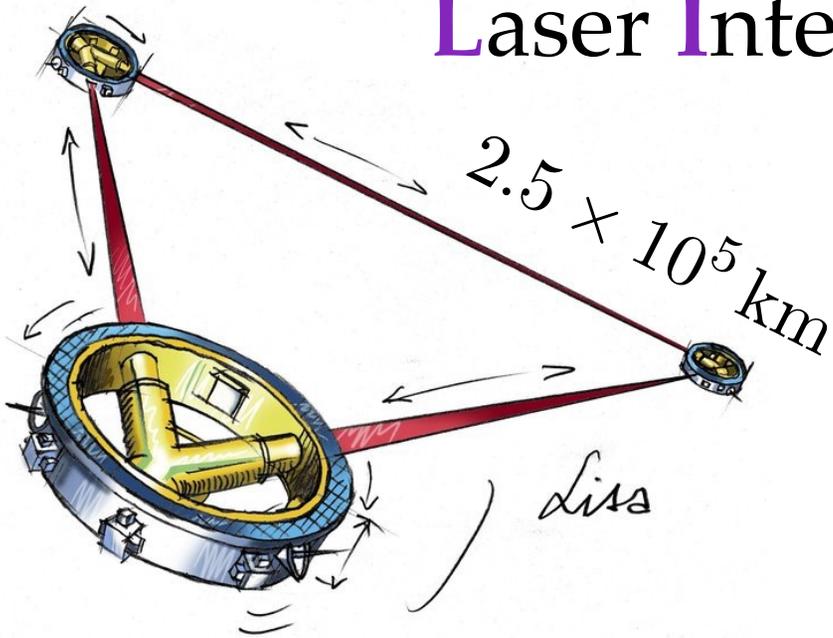
$$\rho_{\text{GW}}^{\text{today}} \approx 0.1 (H_{PT} \Delta t_{PT})^2 \rho_{\gamma} \approx 10^{-5} - 10^{-2} \rho_{\gamma}$$

< CMB  $N_{\text{eff}}$  bounds

$$\omega_{\text{GW}}^{\text{today}} \sim \omega_{\text{GW}} \left( \frac{T_{\text{CMB}}^{\text{today}}}{T_{PT}} \right) \gtrsim \text{mHz} - \text{Hz}$$

# GW detectors

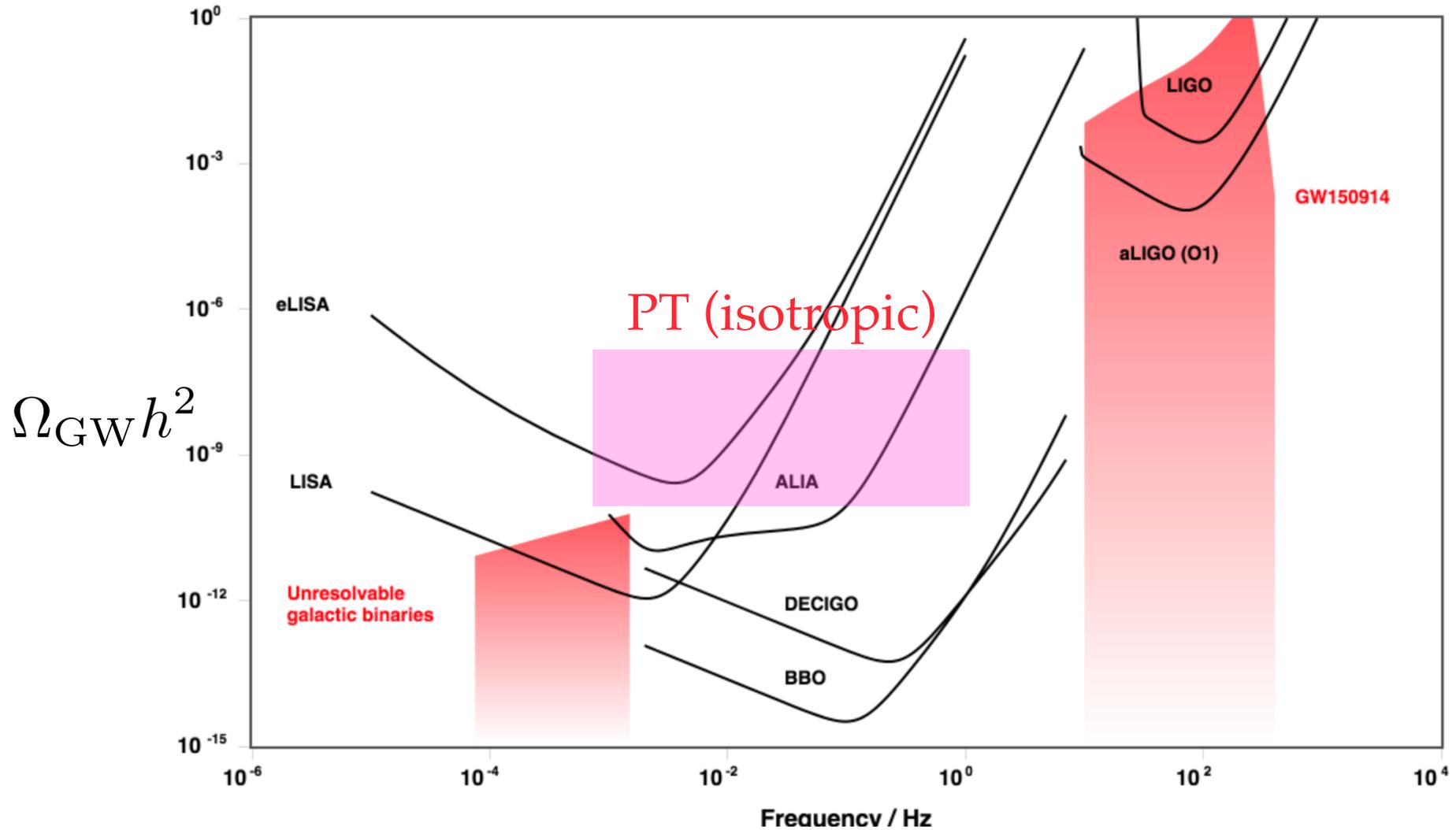
## Laser Interferometer Space Antenna



Similar idea, more satellites, more futuristic

BBO, DECIGO, ALIA  
plus atomic interferometry: MAGIS

# Energy density of GW from PT



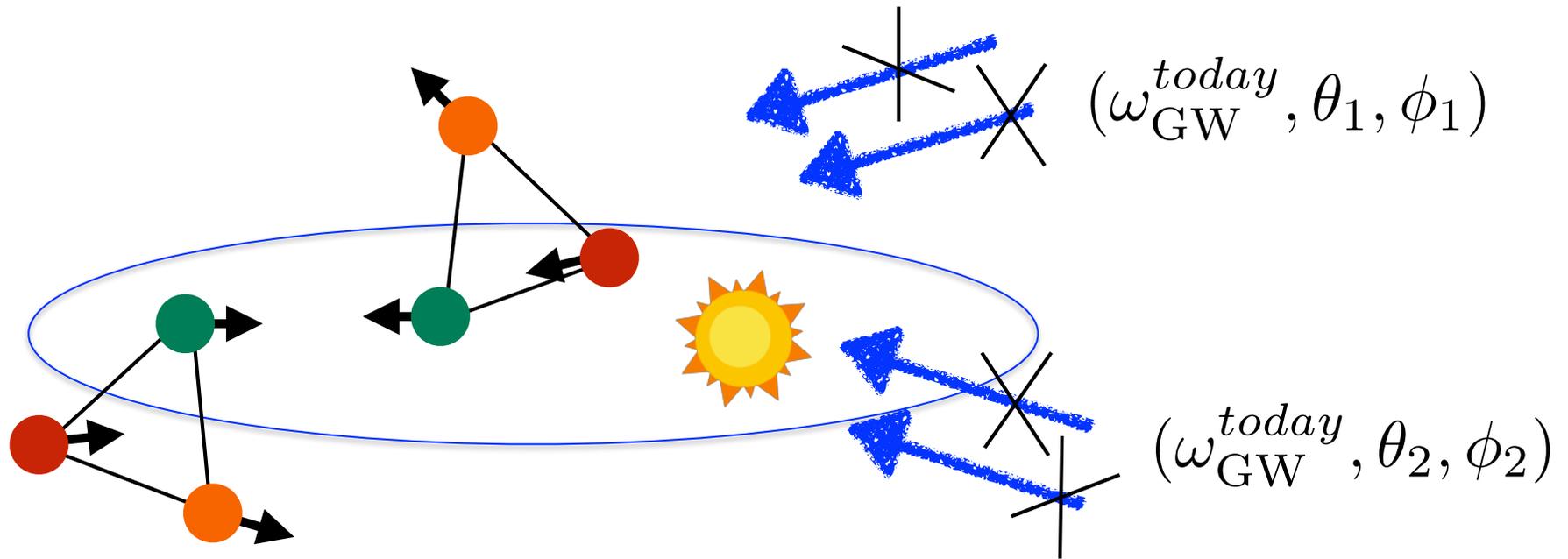
$\Omega_{\text{GW}} h^2$

Frequency (Hz)

[rhcole.com/apps/GWplotter/](http://rhcole.com/apps/GWplotter/)

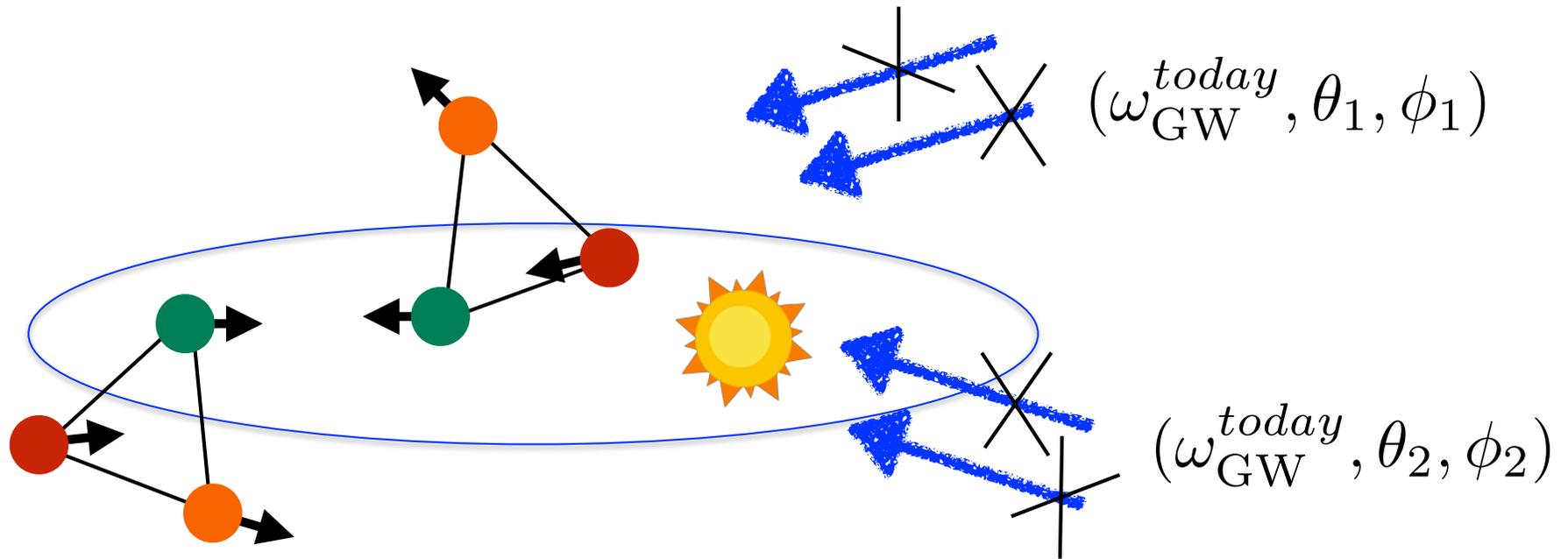
Seeing the anisotropic pattern

# Angular measurement



**Method:** variation of strains in time for each **polarization** mode with different **detector location** / **doppler shift**

# A deconvolution problem



$$\tilde{C}(t, f) = \frac{1}{4\pi} \sum_{\ell m} [p_{\ell m}^E(f)]^* a_{\ell m}^E(f, t)$$

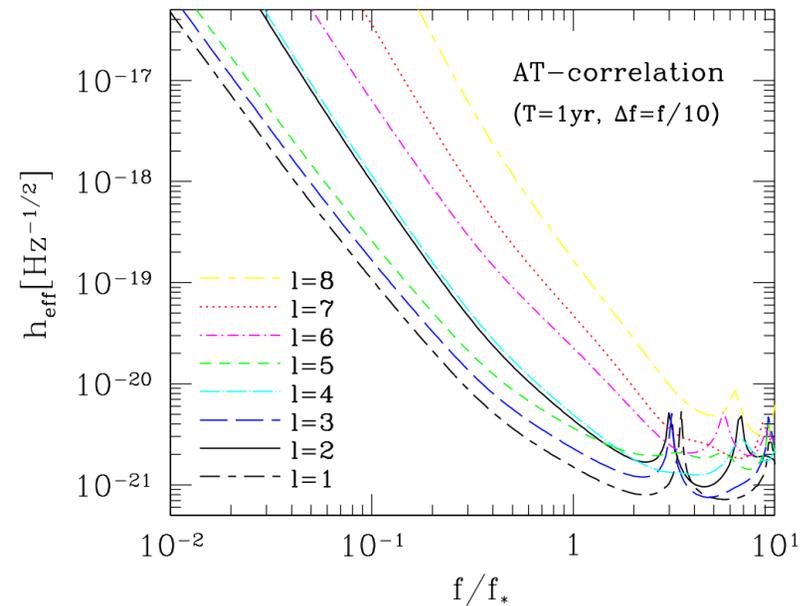
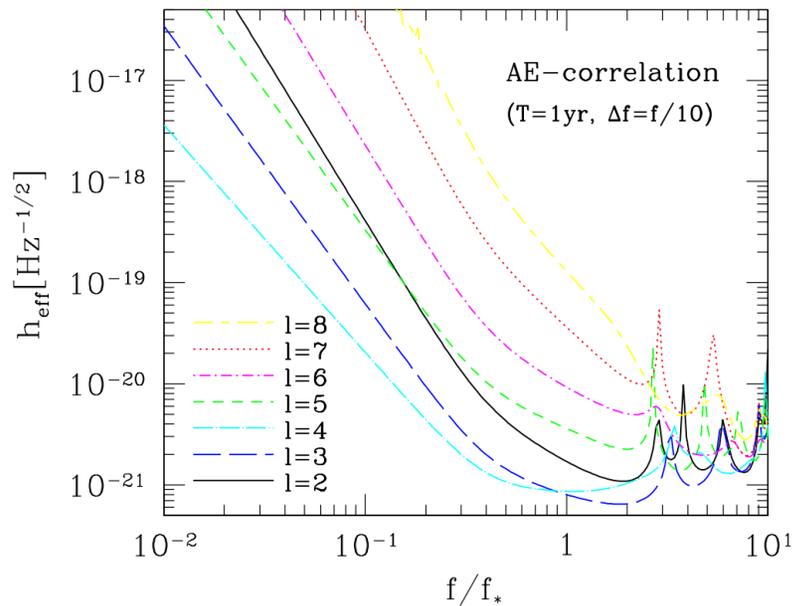
output data

GW background

antenna pattern

# Angular measurement

Projected sensitivity for LISA, Kudoh & Taruya (2005)



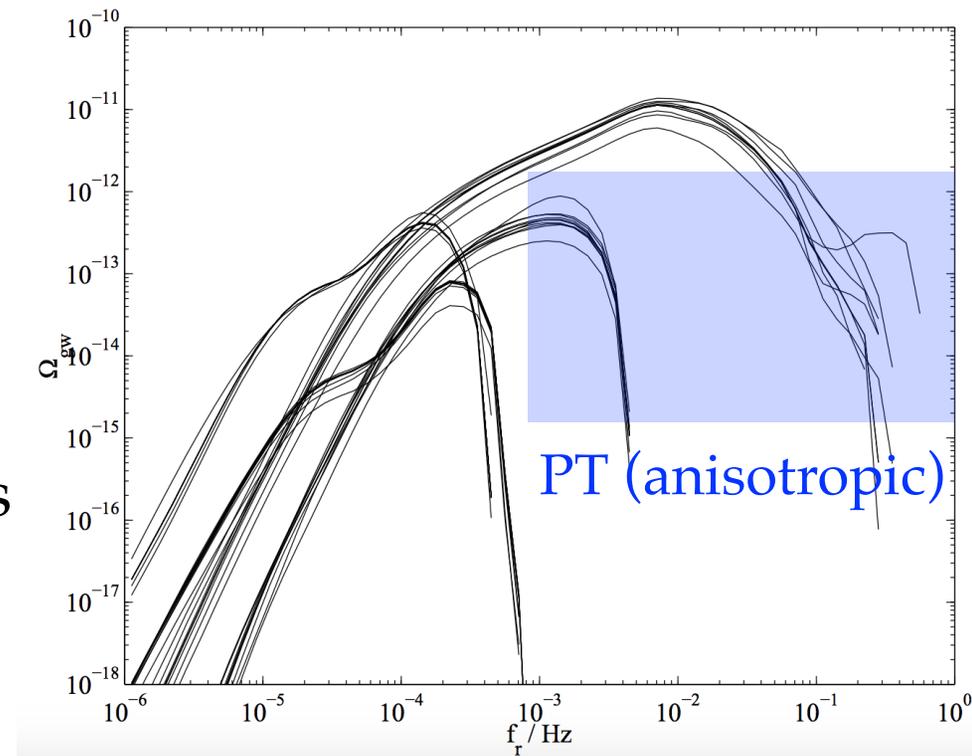
Using cross correlation between different phase readout, **LISA** may get to  $\ell_{\text{max}} \sim 10$ , more detectors (**BBO / DECIGO**) can do much better [e.g., Cutler & Holz (2009)]

# Astrophysical foreground

**Unresolvable white dwarf merger**  
generates the dominant  
background to our signal

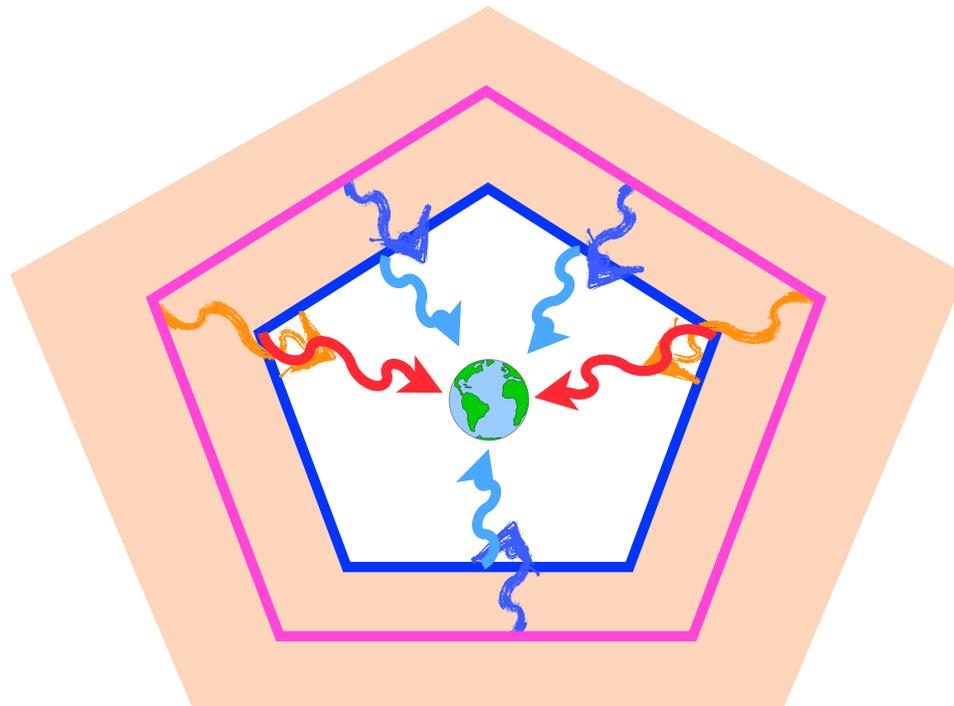
However, most of these backgrounds  
**follow galaxy distribution** and can  
be subtracted with enough data

Adams & Cornish (2013)



Farmer & Phinney (2003)

# Anisotropic GW: minimal story



# Anisotropic Signal

- Natural to have anisotropic GW signal (like CMB)

$$\rho_{\text{GW}}(\theta, \phi) = \bar{\rho}_{\text{GW}} + \delta\rho_{\text{GW}}(\theta, \phi)$$

- Two-point correlators of signal fluctuations

GW-GW

$$C^{\text{GW}}(\theta) \equiv \frac{\langle \rho_{\text{GW}}(1) \rho_{\text{GW}}(2) \rangle_{\theta}}{\bar{\rho}_{\text{GW}}^2}$$

GW-CMB

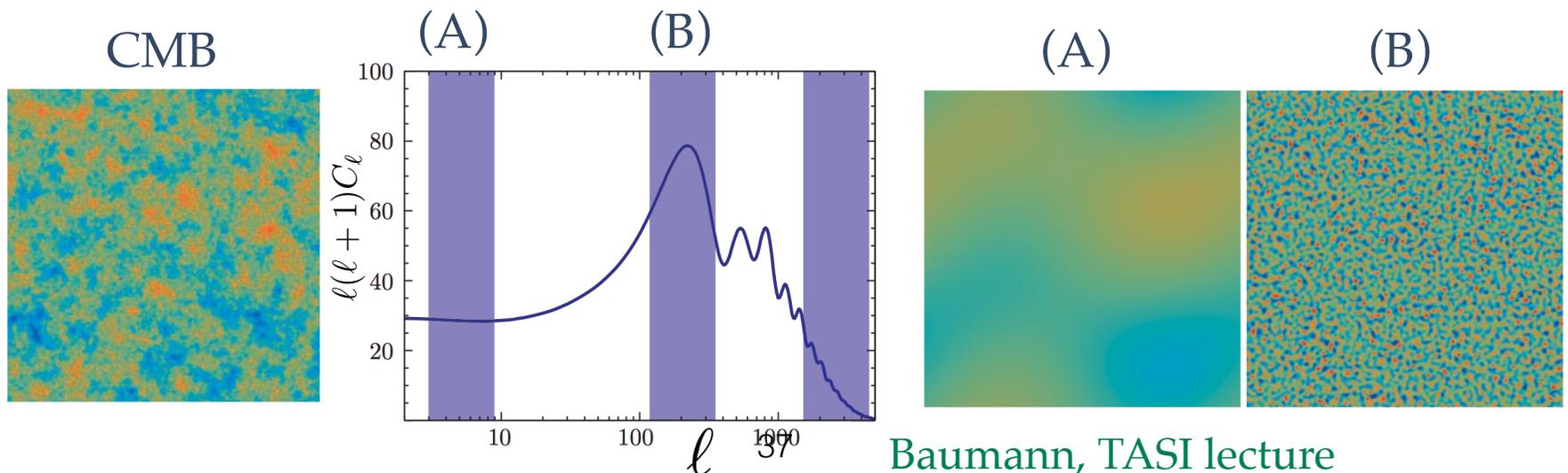
$$C^{\text{cross}}(\theta) \equiv \frac{\langle \rho_{\text{GW}}(1) \rho_{\text{CMB}}(2) \rangle_{\theta}}{\bar{\rho}_{\text{GW}} \bar{\rho}_{\text{CMB}}}$$

# In order to see the anisotropy

$$C^{\text{GW}}(\hat{n}) = \sum_{\ell m} C_{\ell m} Y_{\ell m}(\hat{n})$$

$\sqrt{C_\ell} >$  detector sensitivity

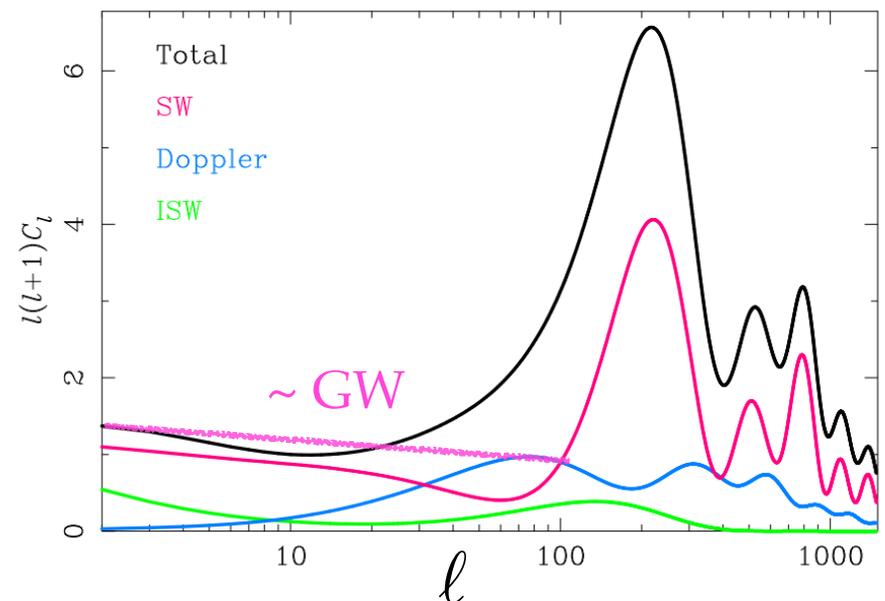
Angular resolution  $< \pi \ell_{\text{max}}^{-1}$



# Minimal Story

- Single source of primordial perturbations (= quantum fluctuations in inflaton field)
- GW anisotropy is totally correlated with primordial photon perturbation
- Roughly scale-invariant primordial perturbation:

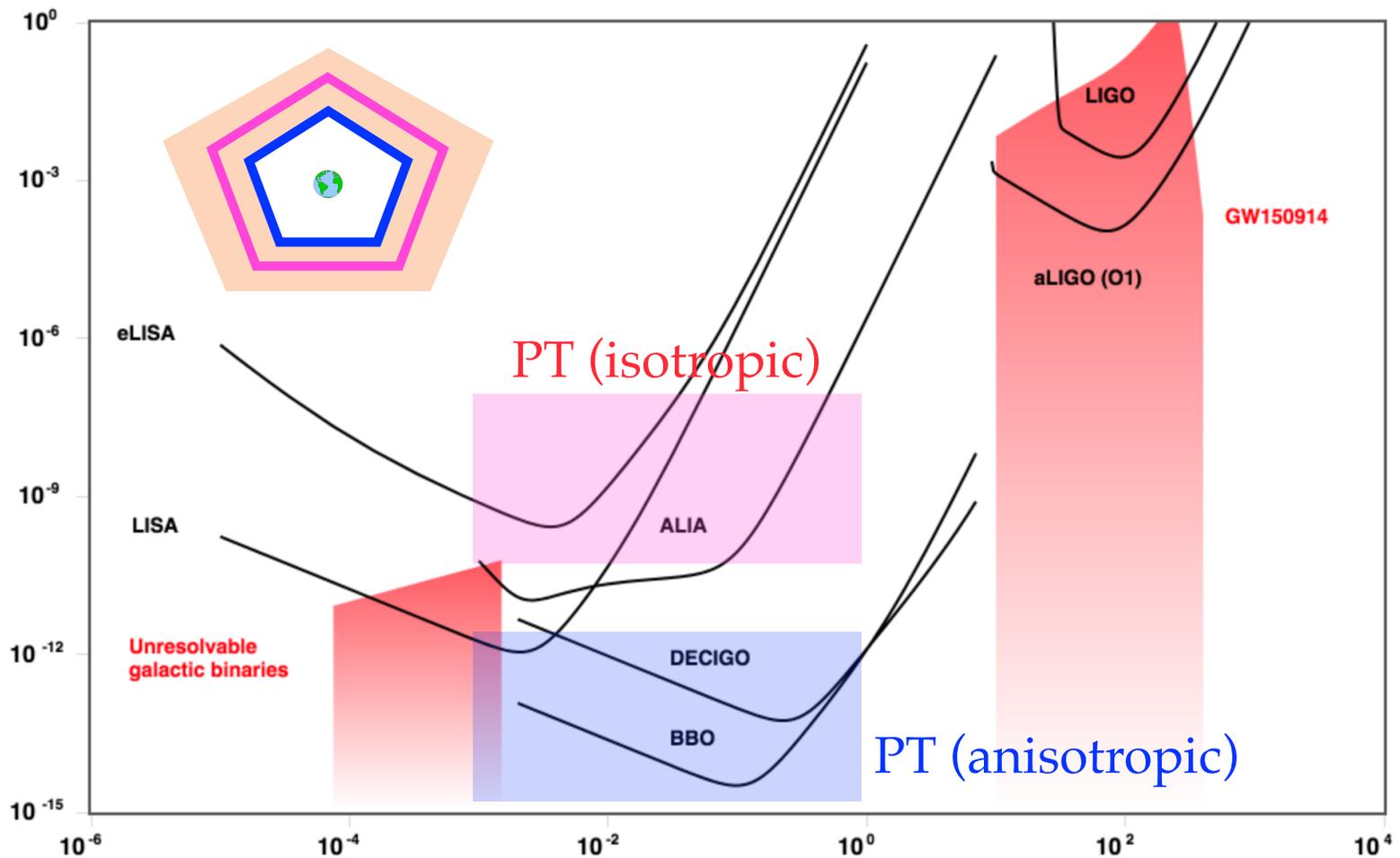
$$\left( \frac{\delta\rho_{\text{GW}}}{\rho_{\text{GW}}} \right)_\ell \sim \sqrt{C_\ell^{\text{GW}}} \sim \frac{10^{-5}}{\ell}$$



# Detection possibility

$$\delta\rho_{\text{GW}}^{\text{today}} \approx 10^{-10} - 10^{-7} \rho_{\gamma}$$

$\Omega_{\text{GW}} h^2$



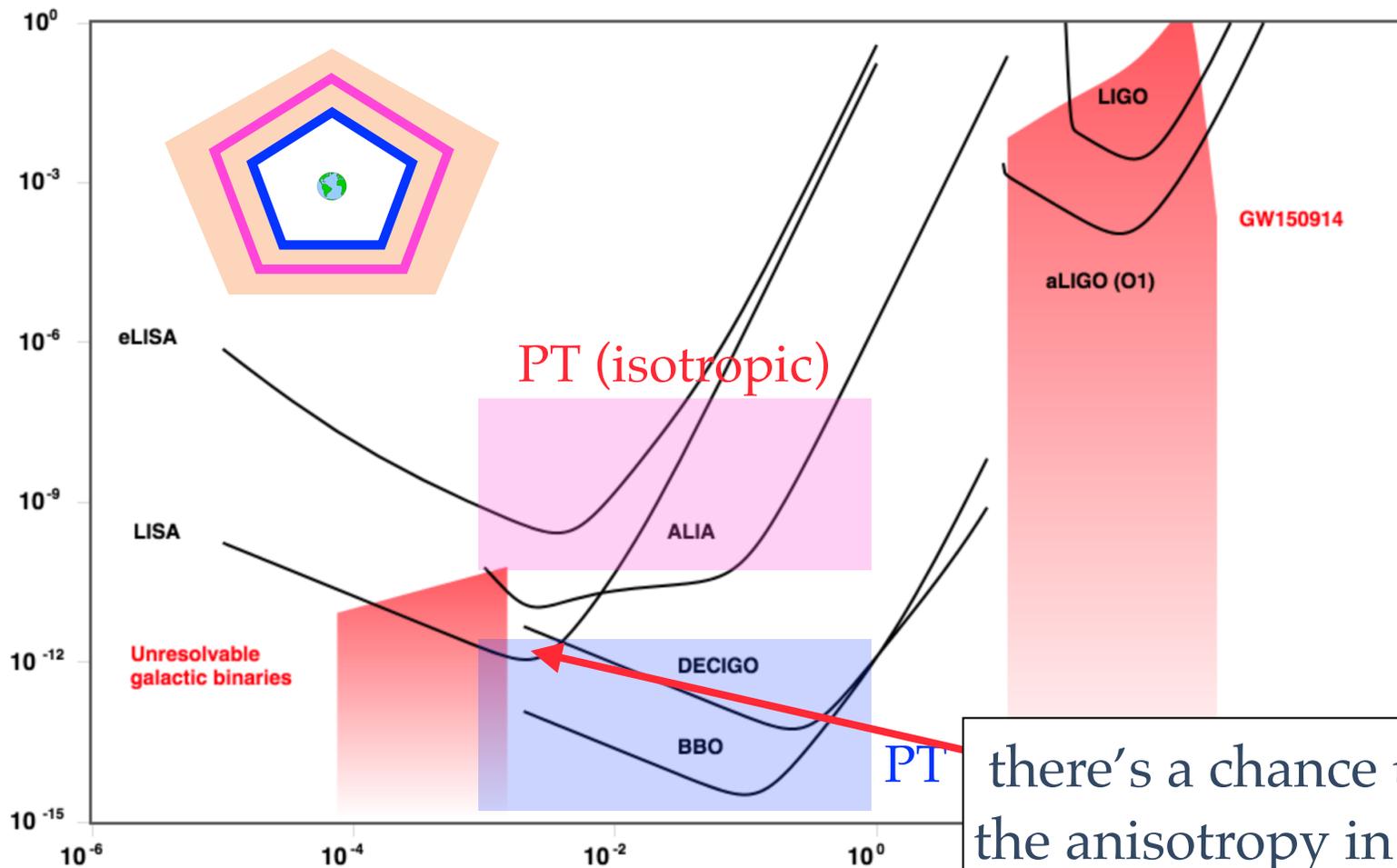
Frequency (Hz)

[rhcole.com/apps/GWplotter/](http://rhcole.com/apps/GWplotter/)

# Detection possibility

$$\delta\rho_{\text{GW}}^{\text{today}} \approx 10^{-10} - 10^{-7} \rho_{\gamma}$$

$\Omega_{\text{GW}} h^2$



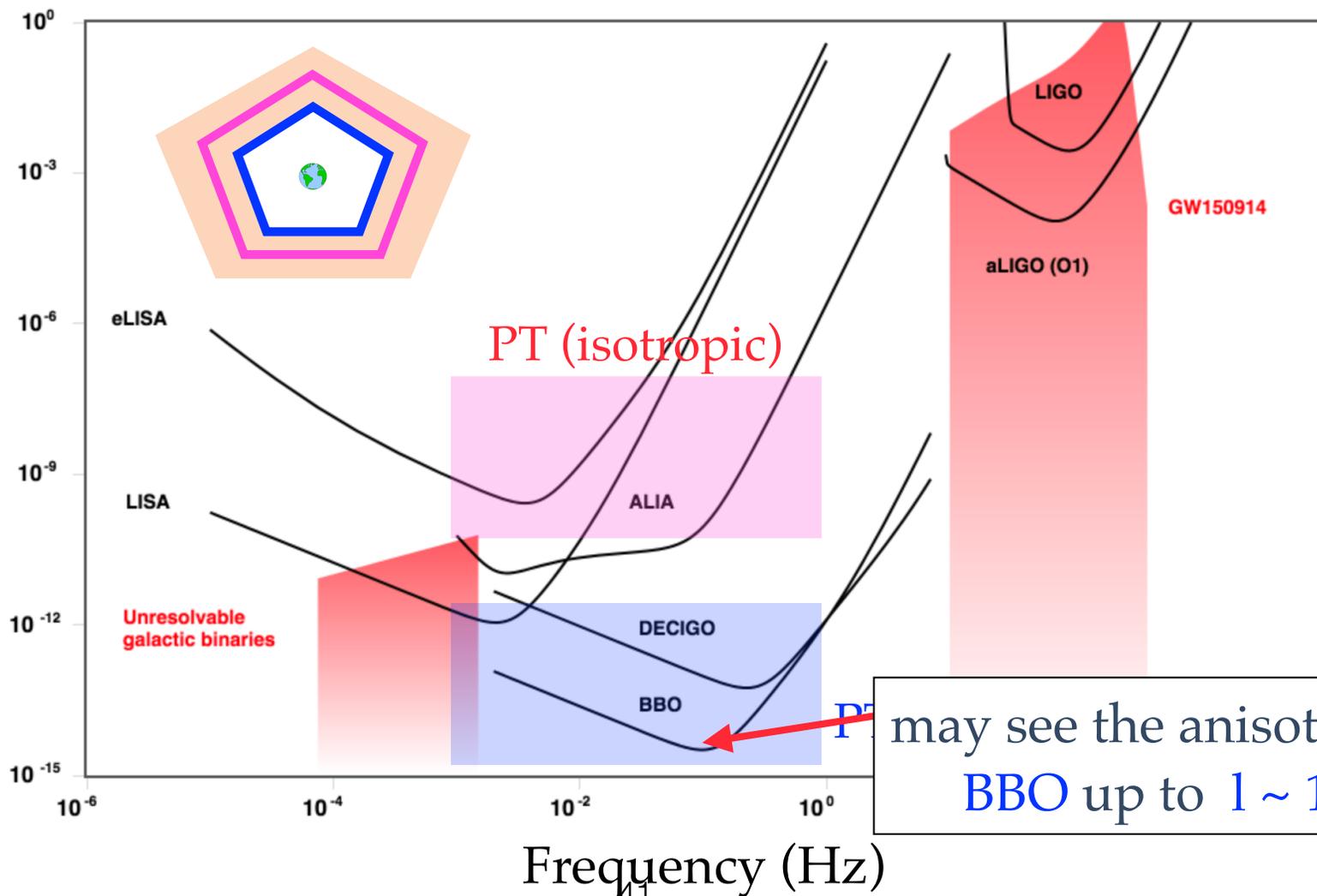
there's a chance to see the anisotropy in **LISA**!

Frequency (Hz)

# Detection possibility

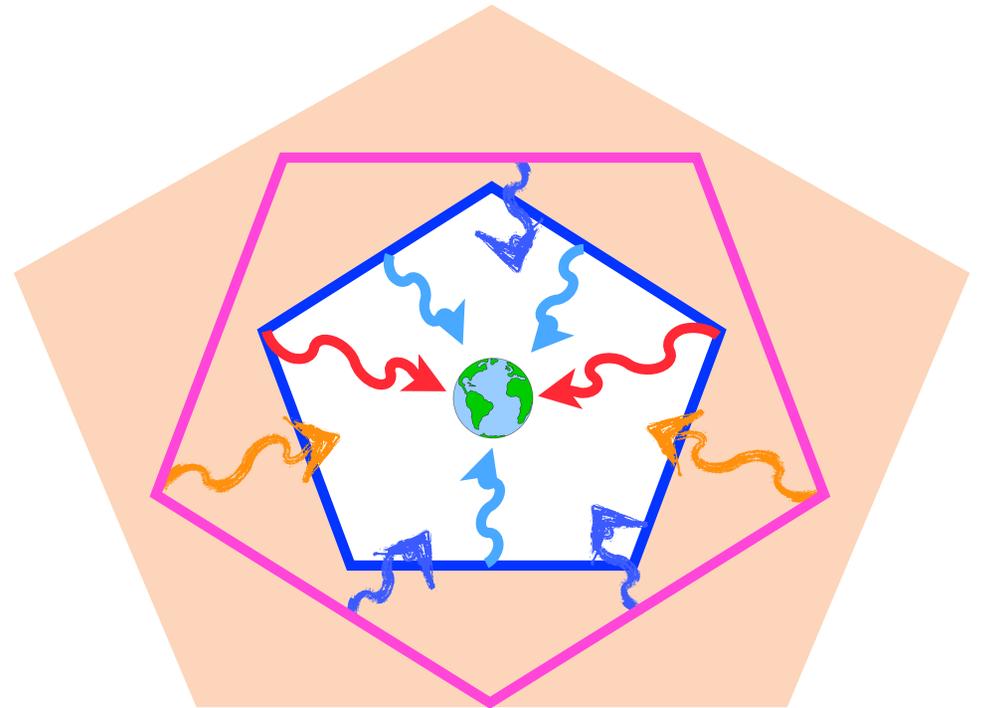
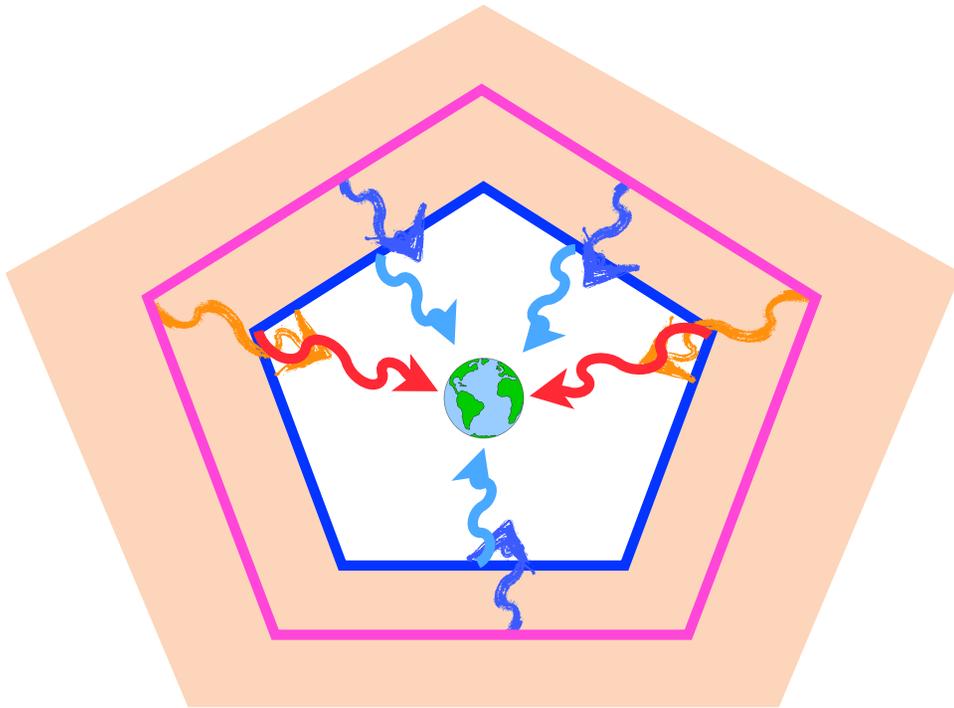
$$\delta\rho_{\text{GW}}^{\text{today}} \approx 10^{-10} - 10^{-7} \rho_{\gamma}$$

$\Omega_{\text{GW}} h^2$



may see the anisotropy in BBO up to  $1 \sim 100$  !

# A Non-minimal Story



# Non-minimal story

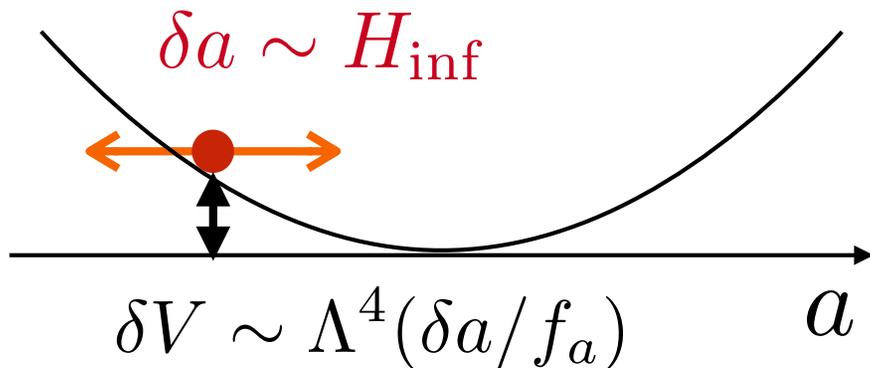
- There could be multiple sources of primordial fluctuations
- The GW and CMB maps are not necessarily correlated

# e.g. a curvaton model

In addition to the inflaton there is an **Axion-Like Particle** with quantum fluctuations during inflation

$$V = \Lambda^4 \left(1 - \sin \frac{a}{f_a}\right)$$

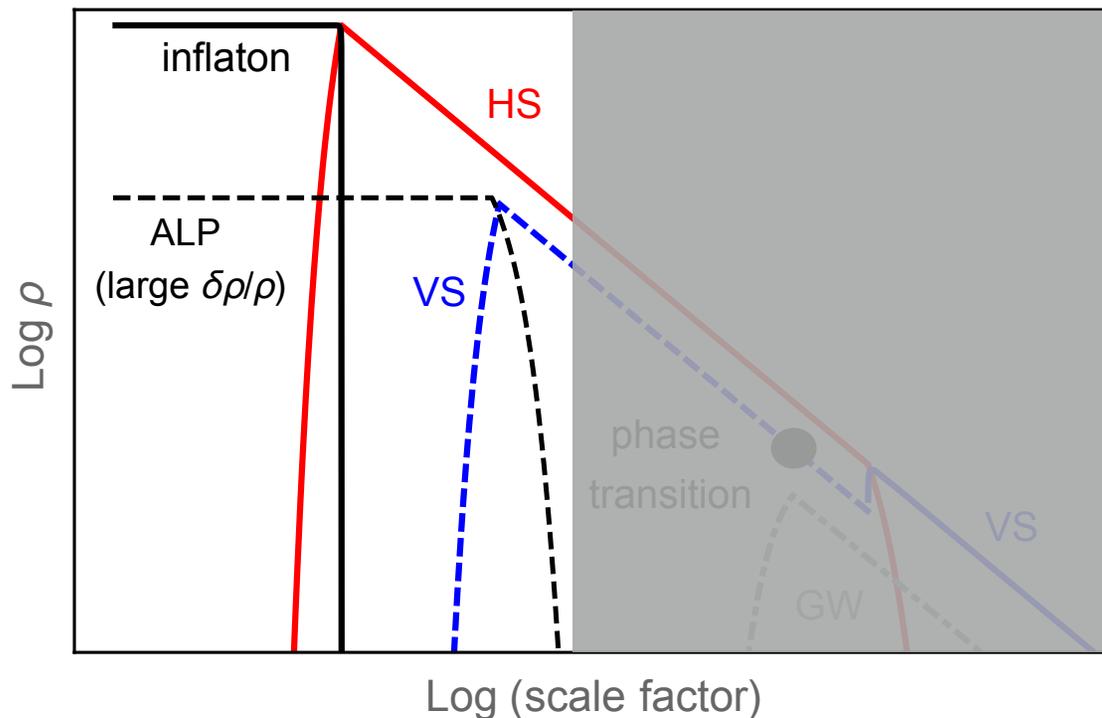
$$\frac{\delta\rho_a}{\rho_a} \sim \frac{\delta V}{V} \sim \frac{H_{\text{inf}}}{f_a}$$



can generate (possibly larger!) uncorrelated perturbations to the inflaton fluctuations

# e.g. a curvaton model

ALP with smaller energy density but larger perturbation decays into visible sector (**VS**) particles, while inflaton decays into a hidden sector (**HS**)



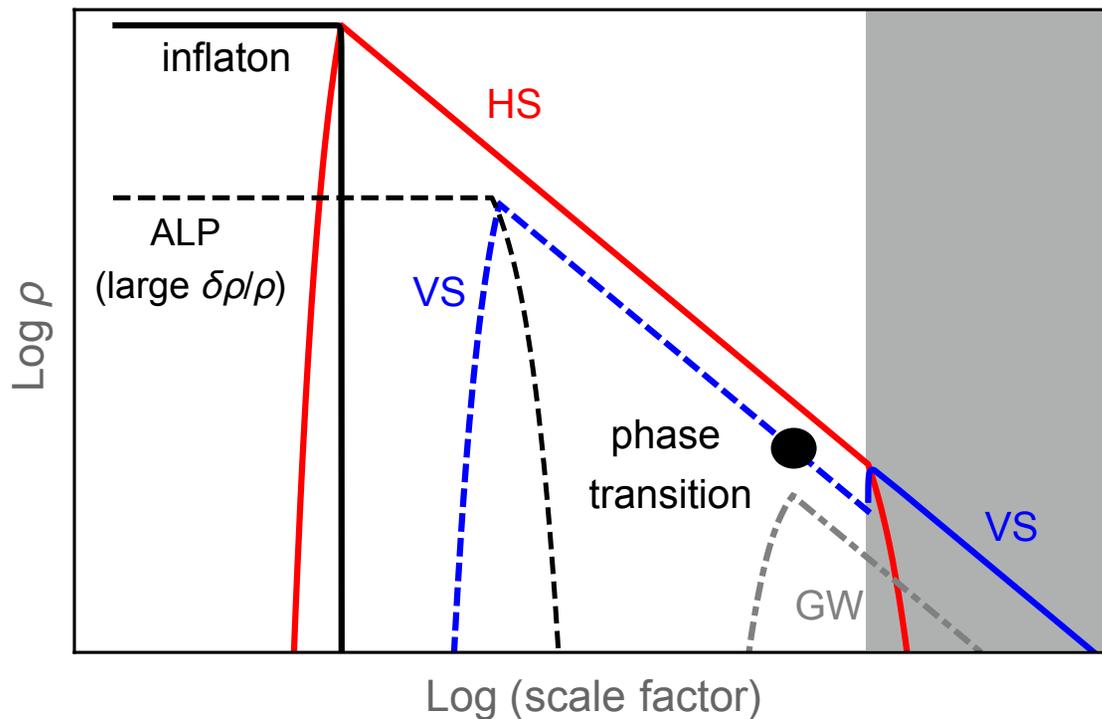
$$\left(\frac{\delta\rho}{\rho}\right)_{\text{HS}} \leq 10^{-5}$$

$$\left(\frac{\delta\rho}{\rho}\right)_{\text{VS}} \geq 10^{-5}$$

remain uncorrelated if  
HS-VS are mostly decoupled

# e.g. a curvaton model

**VS** undergoes a strong first order PT, producing GW with VS perturbation

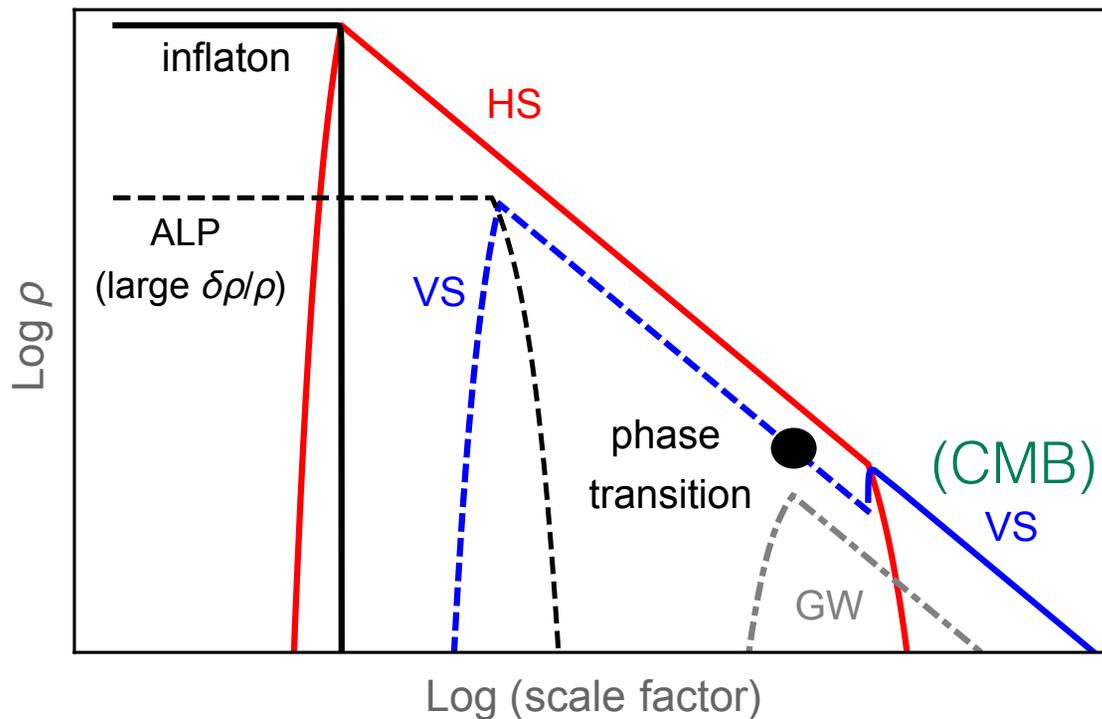


$$\left(\frac{\delta\rho}{\rho}\right)_{\text{HS}} \leq 10^{-5}$$

$$\left(\frac{\delta\rho}{\rho}\right)_{\text{GW}} \sim \left(\frac{\delta\rho}{\rho}\right)_{\text{VS}} \geq 10^{-5}$$

# e.g. a curvaton model

**HS** decays into **VS**, dominates energy density and suppresses photon perturbation to the observed value



$$\left(\frac{\delta\rho}{\rho}\right)_{\text{CMB}} \sim \left(\frac{\rho_{\text{VS}}}{\rho_{\text{HS}}}\right) \left(\frac{\delta\rho}{\rho}\right)_{\text{GW}} + \left(\frac{\delta\rho}{\rho}\right)_{\text{HS}}$$

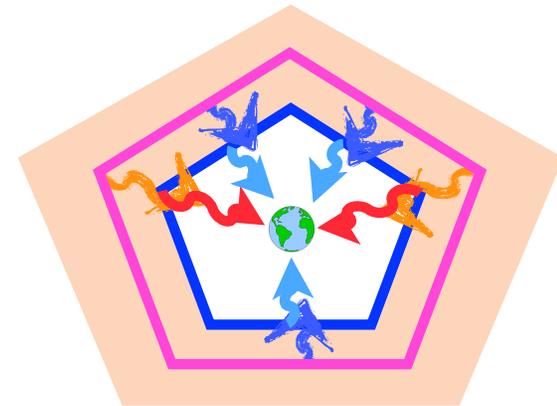
before HS decay

$$\sim 10^{-5}$$

# Correlated GWB & CMB

If density perturbation is dominated by the **1st** term

$$\left(\frac{\delta\rho}{\rho}\right)_{\text{CMB}} \sim \boxed{\left(\frac{\rho_{\text{VS}}}{\rho_{\text{HS}}}\right) \left(\frac{\delta\rho}{\rho}\right)_{\text{GW}}} + \left(\frac{\delta\rho}{\rho}\right)_{\text{HS}} \sim 10^{-5}$$



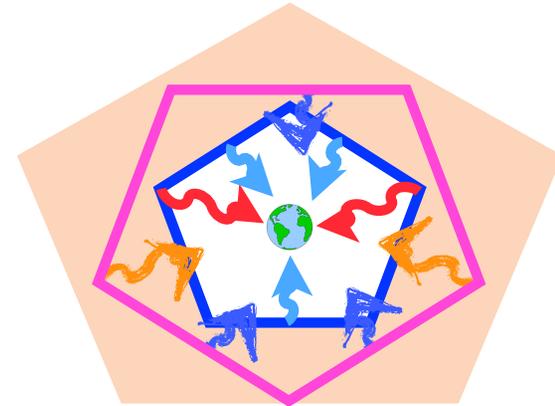
the CMB and GW background are completely **correlated**

$$C^{\text{cross}} \equiv \frac{\langle \rho_{\text{GW}}(1) \rho_{\text{CMB}}(2) \rangle}{\bar{\rho}_{\text{GW}} \bar{\rho}_{\text{CMB}}} \neq 0$$

# Un-correlated GWB & CMB

If density perturbation is dominated by the **2nd** term

$$\left(\frac{\delta\rho}{\rho}\right)_{\text{CMB}} \sim \left(\frac{\rho_{\text{VS}}}{\rho_{\text{HS}}}\right) \left(\frac{\delta\rho}{\rho}\right)_{\text{GW}} + \boxed{\left(\frac{\delta\rho}{\rho}\right)_{\text{HS}}} \sim 10^{-5}$$

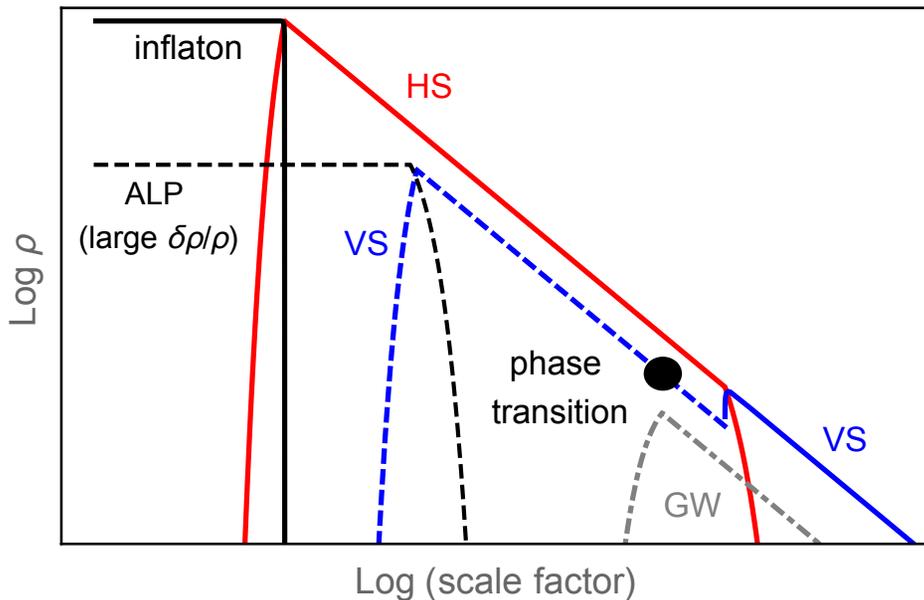


the CMB and GW background are completely **uncorrelated**

$$C^{\text{cross}} \equiv \frac{\langle \rho_{\text{GW}}(1) \rho_{\text{CMB}}(2) \rangle}{\bar{\rho}_{\text{GW}} \bar{\rho}_{\text{CMB}}} = 0$$

# e.g. a curvaton model

$$\delta\rho_{\text{GW}} \sim 0.1 \left( \frac{\rho_{\text{VS}}}{\rho_{\text{HS}}} \right)^2 (H_{\text{PT}} \Delta t_{\text{PT}})^2 \left( \frac{\delta\rho}{\rho} \right)_{\text{GW}} \rho_\gamma < \text{CMB bound on isocurvature}$$



$$H_{\text{PT}} \Delta t_{\text{PT}} = 0.1$$

$$\left( \frac{\delta\rho}{\rho} \right)_{\text{GW}} = 10^{-4} \quad \frac{\rho_{\text{VS}}}{\rho_{\text{HS}}} = 0.1$$

larger energy contrast

anisotropy is visible at BBO up to  $\ell_{\text{max}} \approx 100$

# Conclusion and Outlook

